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AGGREGATE DEMAND WITH PARALLEL MARKETS

ARINDAM DAS-GUPTA
AND
SHOVAN RAY

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SUMMARY

The paper studies aggregate demand in an economy in which households can evade taxes, accumulate illegal wealth and in which there is unaccounted production. The objective is to evaluate the effectiveness of monetary and fiscal policies and also to evaluate anti-black economy policies such as tax enforcement, amnesties and demonetisation. Two regimes, a standard IS-LM regime and a credit-constrained regime, are studied, drawing on a model of household behaviour. It is found that expansionary monetary policy gains in effectiveness relative to expansionary fiscal policy as compared to the all-white economy in both regimes though the former stimulates while the latter curbs parallel production. Increased auditing to detect tax evasion from white factor incomes stimulates the black economy. Amnesties stimulate parallel activity while possibly depressing the white economy.

1. Introduction

The problem of large parallel markets coexisting side by side in many economies has attracted much attention in recent times, both in developed and developing countries. Various policies have been advocated and used in attempts to curb parallel activity. Policies include tax audits and penalties, amnesties and demonetisation. Yet, to our knowledge, no framework exists for the evaluation of the effects of such policies on the parallel economy and on macro activity. Even micro-theoretic evaluation is limited to a subset of such policies, with attention focused on tax audits and penalties.[1] Equally, the efficacy of macro-economic fiscal and monetary policies in the presence of such markets has not been subjected to detailed examination. To our knowledge, the paper by Sundaram and Pandit (1977) represents the only attempt to study the effectiveness of fiscal and monetary policies in the presence of a parallel economy. They, however, do not consider the micro-foundations of household behaviour and are therefore unable to go much further than to point to the reduced effectiveness of macro policies on aggregate demand. Their model is also not equipped to evaluation anti-parallel economy policies.

In this paper we propose a framework for studying the demand side of such economies and the effectiveness of the various policies referred to above. The paper may be viewed as an attempt to develop the core of a complete model in which aggregate supply is also incorporated. This is currently absent from the model developed. Given the objectives of the paper, we closely follow the standard

IS-LM framework. However, since in constructing the model we have in mind the stylised realities of developing countries[2], a short-run model of a credit-constrained economy in the tradition of the financial repression school started by McKinnon (1973) and Shaw (1973) is also studied.

In section 2, a model of household behaviour in the presence of tax evasion, unaccounted wealth accumulation and factor payments is developed. The purpose of this exercise is to study saving behaviour and certain other features needed for the development of the macro framework. Section 3 develops the aggregate IS-LM framework for both flexible interest and credit-constrained regimes. Section 4 contains a comparative static analysis of the effects of fiscal and monetary policies. In section 5, macro-economic effects of anti-parallel market policies are examined. In section 6 we summarise the main conclusions and indicate the direction in which further work may be undertaken. Appendix I contains a detailed discussion of the model of household behaviour while Appendix II contains algebraic expressions for stability conditions and policy multipliers.

The structure of the proposed IS-LM macro model differs from the standard IS-LM model in two important respects. A portion of the total money supply, held as black wealth balances, is unavailable to meet white money demand. This affects the specification of the white LM equation. In particular, since black saving (dissaving)

leads to an increase (a decrease) in black wealth balances, it enters the LM equation. Secondly, three additional equations are added, providing the link between black and white interest rates and specifying parallel goods and money market equilibrium conditions. Of course, IS and LM equations need to be extended to incorporate the effects of anti-black market policies on household and firms behaviour, a task facilitated by the analysis of household behaviour in the next section.

2. A Model of Household Behaviour with Parallel Markets[3]

The following analysis of household behaviour is intended to consistently derive household saving equations and government revenue equations for use in the macro-economic model presented in the next section.

Households live for one period, starting life with initial black and white bequests. They receive income in a large number of small parcels, each with an identical and independent probability of detection if not declared to tax authorities. There also exist firms which indulge in white and black production. The government undertakes income scrutiny of households and imposes penalty on white factor income which is received but not declared. All factor receipts from black production and the part of factor income which is received from white production but is neither declared not detected constitute black income of households. Black income and wealth are subject to wealth scrutiny. If detected, such funds are confiscated.

Household lifetime consumption opportunity are composed of the funds received (bequest and disposable income) from white and black sectors, less the amount they save. What they in fact consume depends not only on the maximum opportunity they have, but also their preference pattern, including in the latter the utility they receive from the bequest they make. These utility considerations in turn are influenced by a number of factors the household may consider important, such as the rate of interest, incomes received, etc. These considerations relating to the micro foundations of household behaviour are discussed in detail in Appendix I.

For the macro-economic analysis in this study, we need to consider the aggregate 'leakage' in the economy for equilibrium in the goods market, following the standard Keynesian method. Note that leakage consists of household savings as well as collections realised by the government. While the details of the derivation of the 'leakage function' are discussed in Appendix I, we note here the basic equation to be used in the macro model:

$$\underset{\quad + \; + \; + \; - \; - \; + \; - \; -}{L = L\,(r,\ Y,\ y;\ W,\ w,\ tf,\ P,\ V)} \qquad (1)$$

In equation (1), the signs underneath the arguments of the leakage function refer to the corresponding partial derivatives. The arguments in the function respectively refer to the white interest rate (R), white income (Y), black income (y), white wealth (W), black wealth (w),

the penal rate of tax upon detection (tf), the probability of escaping detection (P), the fraction of black income and wealth remaining after detection and confiscation (V) and amnesty options allowed by the government (A). The penal rate of tax, tf, is composed of the tax rate (t) and a multiplicative penalty factor ($f > 1$). The assumption of a large number of income parcels with independent probabilities of detection if not reported, implies that ex-ante income equals ex-post income with unit probability. Thus households report none of their income to tax authorities since expected income is maximised by evasion. Consequently only the penal tax rate affects household behaviour. The normal tax rate does not enter as a separate argument.

An additional behavioural equation playing a role in the macro-economic model to be studied is the black saving function. The manner in which this is generated is discussed in detail in Appendix I. For the macro-economic model, we represent it as

$$\underset{\quad + \; + \; - \; - \; + \quad + \; + \; -}{s = s(r,\ y;\ W,\ w,\ tf,\ P,\ V, A)} \qquad (2)$$

One assumption plays an important role in the derivation of the illegal saving function. Given identical (post-tax and penalty) interest rates in both sectors, households are assumed to prefer to save out of white income. They save out of black income only if their total desired saving exceeds their white disposable income. This explains the apparently paradoxical positive effect of the penal tax rate on illegal saving. Since a penal tax rate increase only affects undeclared but detected

(and therefore white) income, households compensate for the excessive fall in white saving by increasing black saving. Other signs in (1) and (2) accord with a priori expectations.

A final relationship obtained from the household model is a link between the white and black interest rates. The relationship is given by

$$r = XR,\ X = (1-P)(\blacksquare-tf)+VP < 1 \qquad (3)$$

The relationship may be explained as follows. XR is the effective (after tax and penalty) white interest rate. Since households are free to vary their saving out of black and white income, they allocate their saving so as to take advantage of the highest effective interest rate. This will mean that no supply of saving will be available in the sector with the lower effective interest rate. Consequently effective interest rates will adjust till they are equal.

3. A Model of Aggregate Demand with Parallel Markets

We now develop a model of aggregate demand by extending the text book IS-LM model so as to incorporate parallel production and tax evasion. The usual case of a fixed money supply flexible interest rate regime is first considered. Modifications are then made to permit the study of a controlled interest rate regime as the latter is of some interest to many developing countries.

(a) The flexible interest rate regime

(i) Preliminaries: The model to be constructed has four asset markets, namely, the markets for black and white money balances and black and white bonds. The model also has two goods markets, one each for white and black goods. In the usual all-white textbook model, one asset market, the bond market, is dropped in view of the stock budget constraint facing agents in the economy and Walras' Law. In the current, parallel market model two independent budget constraints, one each for white and black asset balances have to be met. The independence is a result of the assumption that assets recorded in official books of account cannot, for the economy as a whole, be exchanged for unrecorded assets and vice versa. That is, no net laundering of black funds is assumed to take place. In consequence, two asset markets, one black and one white, need not be explicitly taken note of. The model has only four independent market equilibrium conditions, two each for money markets and goods markets.

The exclusion of net laundering does not preclude gross laundering (or the swapping of black and white asset balances between households) by appropriate book entries. An example of such a transaction is the following.

The popular method of dummy loans from friends or relatives works as follows. A black wealth holder persuades a relative to sign a document showing a loan made to the wealth holder by the relative without any

money changing hands (except perhaps a payment for services). The black wealth holder now uses his black wealth in white markets. However, note that the relative's white books must also show the loan given which forces her to conceal an equivalent sum of cash. No net laundering has taken place.

In fact, the laundering rate, or the payment made per unit of funds laundered by the black money holder, is the price which clears black asset markets, given the fact that black and white interest rates are linearly related as has been argued in section 2.

Thus, in the model to be specified, goods markets are cleared by the usual income adjustment mechanism, the white money market is cleared by the white interest rate and the black money market is cleared by the laundering rate.

(ii) Behaviour of firms: In contrast to the household sector, firm behaviour is treated in a rudimentary way. It is helpful to visualise production taking place in two 'rooms', one containing capital goods whose purchase is recorded in official books, which are financed by white borrowing or equity, and the other containing unaccounted capital goods financed by black household funds. White production and black production can be correspondingly distinguished. Unaccounted production facilities are subject to detection and penalty.[4] Unaccounted investment is undertaken to take advantage of the lower black interest rate to the extent that this offsets expected penalties.

In accordance with this parable, firm behaviour is assumed to be summarised by a pair of investment functions, one each for black investment (i) and white investment (I). The simplest forms, $i(r,V)$, $i_r < 0$, $i_V > 0$ and $I(R, tf, P)$, $I_R < 0$, $I_{tf} < 0$, $I_P > 0$, are assumed for these functions. The term V is included in the black investment function since increased wealth scrutiny decreases the value of black factor payments relative to white factor payments. Thus firms will have to pay higher wages relative to the white sector which will reduce profitability and investment. Similar considerations lead to the dependence of white investment on the penal tax rate and the probability of escaping audit.

Before leaving this section, one remark may be worth making. Although it is normal to treat R as equal to the return on government bonds in equilibrium, this may no longer be true of evasion. It is reasonable to suppose that interest payments on government bonds are not as easily evaded as returns on private sector investment. Thus, in equilibrium, the pre-tax return on investment will be lower than the return on government bonds. Evasion, therefore, may increase the cost of debt finance through bonds in addition to lowering tax receipts.

(iii) <u>The circular flow of income and equilibrium in goods markets</u>: The circular flow of income in the model is shown in Figure 1. The flow diagram extends the textbook diagram of a circular flow with producers, households and government to incorporate black income and expenditure. While the diagram is

largely self-explanatory, one point needs clarification. Since firms and households maintain separate books of accounts, an expenditure out of accounted funds with households need not be shown as a receipt on the books maintained by firms and vice versa. For example, white consumption expenditure by households need not equal sales receipts on consumption goods sold by firms and shown on their books. Of course, total black and white consumption expenditure (or investment expenditure) will equal total receipts from sales of consumption goods (or investment goods). There are three exceptions to this. First, factor payments not accounted for on the books of firms cannot be shown as income on household books. Secondly, black investment must be financed entirely out of black household saving since firms borrowing black household funds will not be in a position to reveal the source of these funds on their books and, conversely, funds borrowed from white markets have to be accounted for in the books of firms.[5] Finally, government expenditure must clearly match firms' receipts from such expenditure.

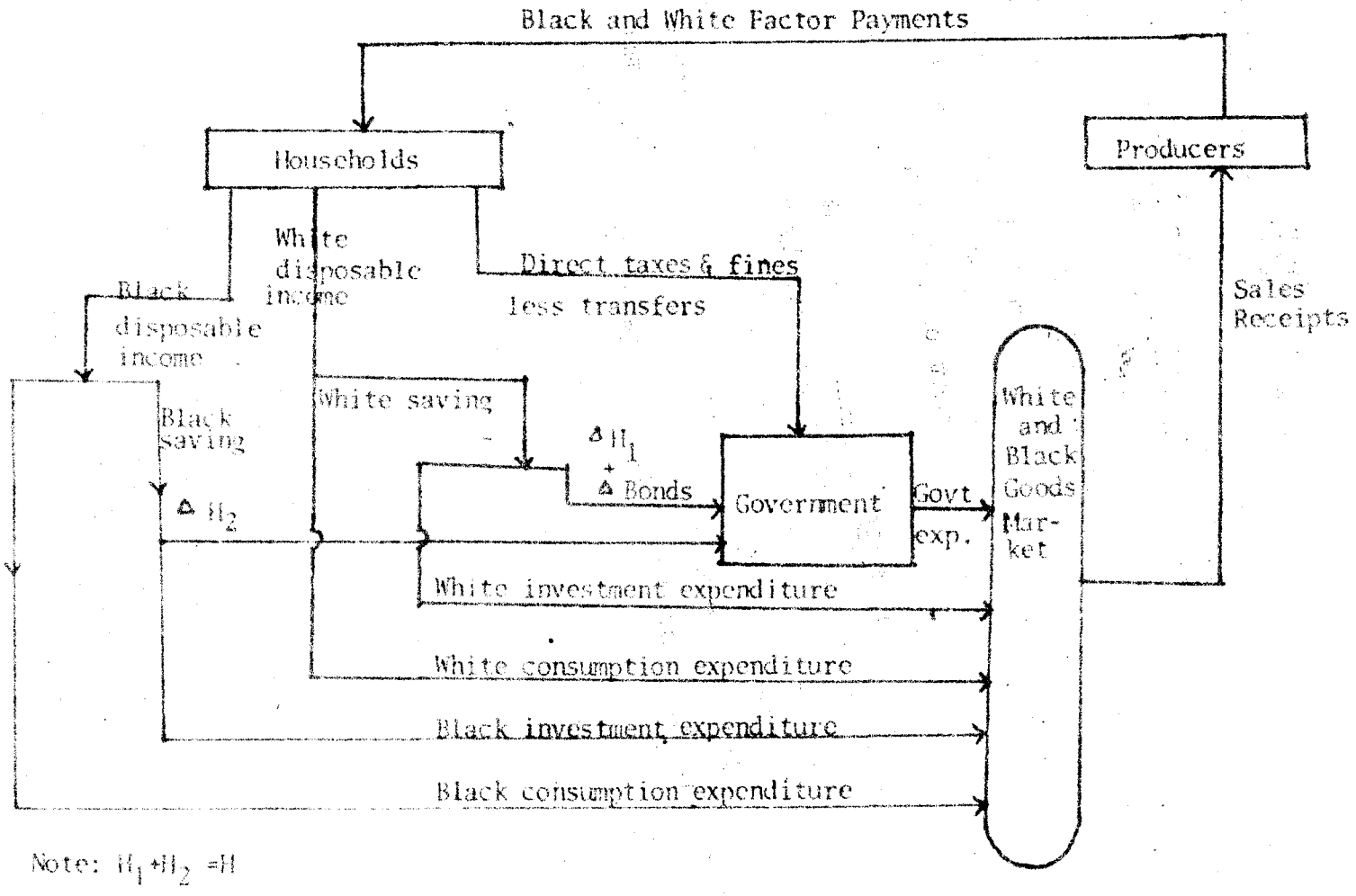

Figure 1. Circular Flow of Income for an Economy with Evasion and Parallel Production

A consequence of this lack of agreement is that expenditure on white goods need not equal white production and similarly for black production and expenditure. However, such discrepancies will be random and, in expected terms, production and expenditure will be equal in both sectors. We may therefore still examine expected levels of output. It will be understood that expected or average levels of black and white output are being studied below.

The upshot of the discussion above and in section 2 is that the following goods market equilibrium conditions can now be specified.

$$\underset{- \quad - +}{G+I(R)+i(r,V)} = \underset{+ + + - - \quad + - -}{L(r,Y,y;W,w,tf,P,V)} \qquad (4)$$

$$\underset{- +}{i(r,V)} = \underset{+ + - - \quad + + + -}{s(r,y,W,w,tf,P,V,A)} \qquad (5)$$

Equation (4) is the economy-wide IS equation for both black and white markets, while equation (5) is the 'is' equation for black goods market equilibrium. G represents government expenditure.

(iv) Money market: Money demand by firms and households has not been formally modelled since the main interest in the household model is to derive the leakage function, the black saving function and the black-white interest rate link while investment demand is the main focus of the discussion of firm behaviour.

Money demand functions of the traditional form are assumed for white money demand (M) and black money demand (m) so that

$$M = M \underset{-}{(R,} \underset{+}{Y)}$$

and

$$m = m \underset{-}{(r,} \underset{+}{y,} Z)$$

In the black money demand function, Z is the laundering rate. The sign of the partial derivative of m with respect to Z is left undetermined as it plays no role in the policy analysis to follow.

Fractional reserve banking is assumed away so that white money supply consists entirely of the monetary base, H, less leakage to the black economy. A convenient way of representing this leakage is the end-of-period black wealth. White money supply is therefore given by H-Vw-s. Correspondingly, black money supply must be Vw+s. The money market equilibrium conditions are therefore

$$M \underset{-}{(R,} \underset{+}{Y)} = H-Vw-s \qquad \text{(white 'LM' equation)} \quad (6)$$

and

$$Vw+s = m \underset{-}{(r,} \underset{+}{y,} Z) \quad \text{(Black 'LM' equation)} \quad (7)$$

(v) The complete model: The complete model is given by the four equilibrium conditions (4), (5), (6) and (7) and the interest rate relation (3). Substituting (3). Substituting (3) into (4), (5) and (6), the model may be seen to consist of a three-equation sub-system jointly determining equilibrium values of R, Y and y and an independent equation (7), which determines the equilibrium laundering rate. Assuming stable adjustment to exogenous displacements, the model can be conveniently graphed in a three-equadrant diagram. This is done in Figure 2.

The economy-wide IS curve (4) is plotted along with the white LM curve (6) in the first quadrant, which has R on the vertical axis and Y on the horizontal axis. The diagram is drawn for a given value of black output, y. A higher value of y would result in a left-ward shift of the IS curve due to increased leakage from white money supply. The intersection of the two curves gives the equilibrium values of white income and the white interest rate for the given level of black output.

The second quadrant graphs the relation r=XR from equation (3) and determines the black interest rate given the white interest rate. The third quadrant has the black goods market equilibrium locus of r and y values. Starting from any given equilibrium values of Y and R, equilibrium values of r and y can be determined by following through the dotted line EE in Figure 2 from the first to the third quadrant. Clearly, the value of y in the third quadrant must be consistent with the value of y underlying the loci in the first quadrant

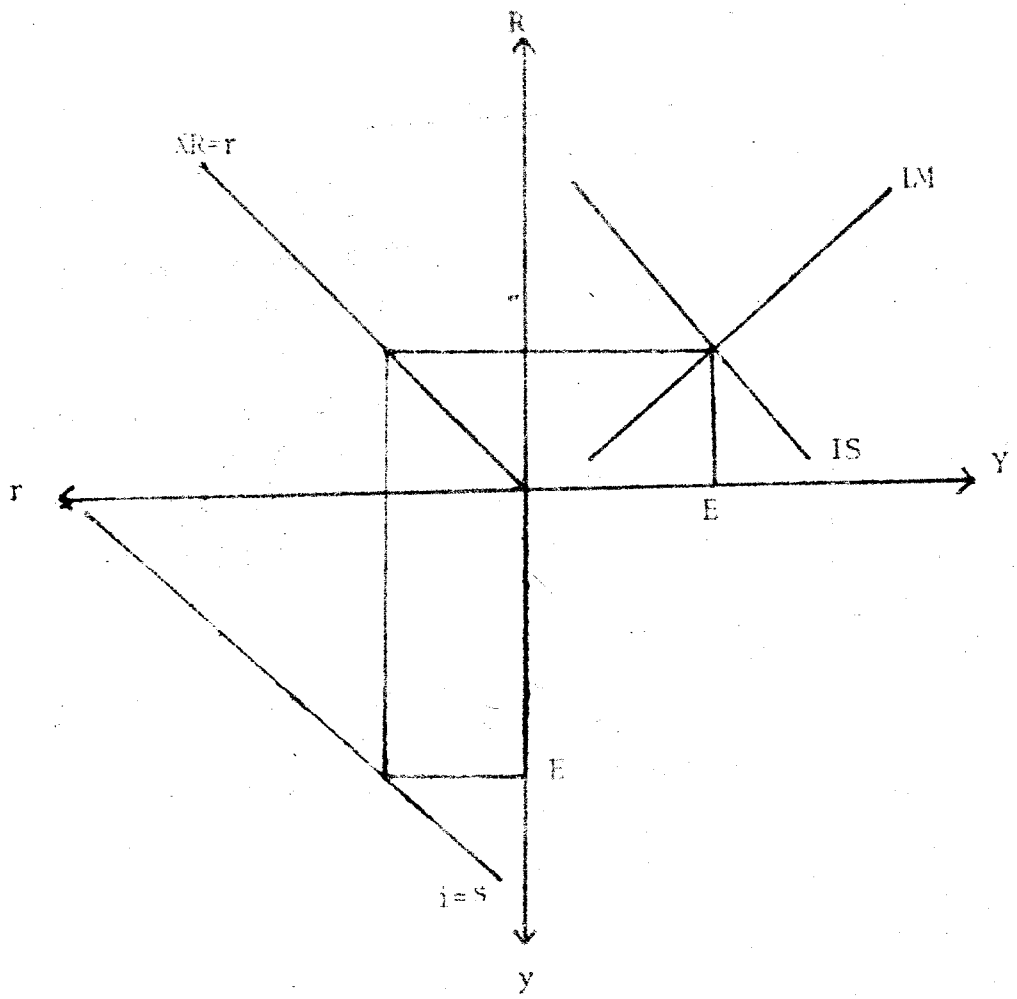

Figure 2. The IS-LM Model with Parallel Production

for this to be an equilibrium. The working of the model is explained more fully in the course of comparative static exercises of the next two sections. Before these exercises are undertaken, the controlled interest rate regime is discussed.

(b) The Credit-Constrained Regime

In India and in certain other less developed economies, formal sector interest rates are controlled and credit is rationed. The reason for such a policy is to provide cheap credit for socially productive investment and also to make cheap debt finance available to the government. The possibly adverse effects of such policy have been the subject of discussion, in the context of a growing economy, chiefly by the financial repression school of McKinnon (1973) and Shaw (1973).[7] The main conclusion of this literature is that such repression depresses saving and promotes overly capital-intensive production in favoured industries. The effectiveness of government policy in the presence of such repression and the effect of anti-black policy in this context are therefore of great interest.

The main alteration required to the model of the previous section is to identify the effects of constrained money supply on investment and its rate of return, given that the government bond rate is also fixed.

It is assumed, in the spirit of McKinnon (1973), that firms require working capital in order to successfully utilise investment goods they contemplate buying . White investment is undertaken only if such finance is granted by the money rationing authority. White investment in this constrained situation is limited to $I(H-Vw-s, tf, 0)$, $I_1 > 0, I_{tf} < 0$, $I_p > 0$, where it is implicitly assumed that increased money supply on account of a decrease in black saving is not mopped up by the rationing authority. This is a version of the celebrated complementarity hypothesis of McKinnon (1973) according to which financial and physical assets are complementary rather than substitutes in a supply-constrained situation.[8] Given constrained investment, the effective white return on investment will exceed the government bond rate after taxes for most types of rationing policies even though many investments which would not be undertaken if the government bond rate had been free to float now find promoters. Furthermore, if the credit constraint is eased, both average and marginal rates of return on investment should decline as more investment is undertaken. Accordingly the equations determining income and the white interest rate are now given by

$$I(H-Vw-s, tf, P)+i(r, V)+G = L \qquad (4')$$

$$R = R(I), \quad R' < 0 \qquad (6')$$

The remaining equations of the first model, (3),(5) and (7), continue to hold.

An interesting property of the credit-constrained regime is the one-way causality from the black market to the white goods market. Given that interest rates are now determined by the money supply and are unaffected by changes in white output, changes in white output do not have any impact on black output.

This completes the description of the aggregate demand model under both regimes. We may now turn to policy analysis.

4. Impact of Fiscal and Monetary Policies

a. Increased Government Expenditure

An increase in government expenditures (with no change in the stock of money or the tax rate) causes and increase in white income and interest rates but a decrease in black income. The chain of causation runs as follows. The increase in government expenditure raises disposable income and aggregate consumption expenditure which initially stimulates white output. In Figure 2 this would be represented by a rightward shift of the aggregate IS curve. However, increased white income raises white money demand which puts upward pressure on white interest rates. As households now start withdrawing saving from the black sector, black interest rates also increase. Increased interest rates crowd out investment, dampening the white income increase and reducing black income.

While the rising black interest rate stimulates black saving, the fall in black income reduces black saving by an amount which more than offsets the increase induced by the rising interest rate. The decrease in black saving leads to reduced leakage from the white money supply resulting in a rightward shift of the white IM curve leading to a further stimulation of white output. After secondary and tertiary effects the new equilibrium configuration is as shown in Figure 3 by the line FF passing through the intersection of the loci IS' and IM'. The initial equilibrium is shown by the line EE.

The effect on total output, Y+y, can be found by adding up algebraic expressions for the individual effects which are listed in Table A-4 of Appendix II. The sign of the total effect is ambiguous. A sufficient condition for increased government expansion to have an overall expansionary effect is that the sensitivity of black saving to unaccounted output (s_y) exceeds the sensitivity of white money demand to white output (M_Y). If this is the case, then white money supply will increase enough to sufficiently dampen crowding out. An interesting implication of this result is that a trade-off between anti-parallel market policy and the effectiveness of fiscal policy may exist. Specifically, we have wealth scrutiny in mind. Wealth scrutiny reduces disposable black income at any given level of black output. Consequently, the sensitivity of black saving to black output will be lowered at any

given level of black disposable income as wealth scrutiny is stepped up. Therefore, in order to maintain the overall expansionary influence of fiscal policy, lenient wealth scrutiny may be needed.[9]

In the credit-constrained regime, the lack of feed-back from the white goods market to the black sector or to the white money market implies that government expenditure has no crowding out effect and, consequently, no effect on black income or any interest rate. The multiplier for white (and total) income is the inverse of the leakage propensity as in the simple "Keynesian cross" model.

An increase in the tax rate has identical effects to an increase in the fine rate and is taken up in the next section.

b. Monetary Expansion

As can be expected, monetary expansion reduces interest rates. The immediate consequence of this is an expansion in output in both black and white sectors. However, if the interest sensitivity of illegal saving is high enough, much of the additional money supply will leak to the black sector. Both the fall in the interest rate and the increase in black income stimulate black consumption demand while having opposing effects on white consumption demand. The upshot is that the initial rightward shift of the IM curve is curbed and a leftward

shift of the IS curve results. The case where the final LM curve coincides with the initial LM curve is shown in Figure 2. The line MM passes through the new equilibrium configuration.

The aggregate effect on (y+Y) must clearly be positive (this is shown formally in Appendix II). This leads to the interesting conclusion that monetary expansion has increased effectiveness as compared to fiscal policy in an economy with parallel markets, though expansionary fiscal policy still stimulates the white sector.

The effects of monetary expansion in the credit-constrained economy are qualitatively identical. The various cases are shown in Table 1.

5. Policies to Curbe the Black Sector

Here, five policies are considered. The three 'fiscal' policies include an increased fine (or tax) rate and increased income or wealth scrutiny. The two 'monetary' policies are demonetisation and amnesties. These are taken up in turn.

a. 'Fiscal' policies to curb black activity

All three fiscal policies lead to an increased differential between white and black interest rates.

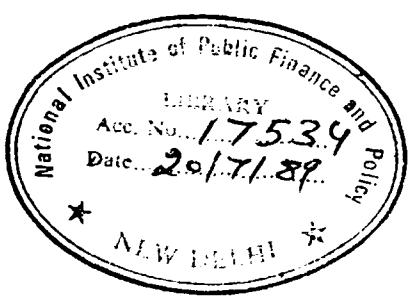

TABLE 1

Effects of Fiscal and Monetary Expansion

Policy	Flexible regime				Constrained regime			
	Y	R	y	(Y+y)	Y	R	y	(Y+y)
G increase	+	+	-	?	+	0	0	+
H increase	?	-	+	+	?	-	+	+

TABLE 2

Effects of Anti-Black Economy Policies

Policy	Flexible regime				Constrained regime			
	Y	R	y	(Y+y)	Y	R	y	(Y+y)
tf increase	?	?	?	?	?	0	?	?
P decrease	?	?	+	?	?	0	+	?
Demonetisation	-	-	?	?	-	-	-	-
Amnesties	?	-	+	+	?	-	+	+

In Figure 3, this is shown by a steeper interest rate ratio line, $r=X_1R$, in the second quadrant. Consequently, at a given white interest rate black investment is stimulated. This stimulative effect conflicts with the normal contractionary effect of increased taxes (or fines) and results in ambiguous macro effects of a change in the tax or fine rate. Similarly, conflicting black saving and investment effects render the effect of increased wealth scrutiny uncertain.

Stricter white income auditing (lower P) will stimulate black production. This is because falling interest rates stimulate consumption at any given level of output, the stimulation being greater for black consumption than for white consumption. Although investment in both sectors is stimulated by falling interest rates, white investment is curbed by stepped up income auditing. Furthermore, unlike the case of raised taxes and fines, the black saving rate at any given income level is reduced on account of stricter auditing. Consequently, the consumption effect is stronger in the case of more severe auditing as compared to raised taxes or fines. The result of this is increased black income following stepped up auditing. This result may be reinterpreted as follows. If white factor payments become harder to conceal, economic activity shifts to the black sector.

When all adjustments are taken into consideration, other effects of stepped up auditing are uncertain. The case where white income decreases on account of a leftward shift in the IS curve to IS', with no net shift in the LM curve, is shown in Figure 3. The new equilibrium values lie on the line PP in Figure 4.

In the constrained regime, the effects on black and white income are qualitatively similar. However, there is no impact of increased taxes or income auditing on the white interest rate. These results are summarised in Table 2.

b. Demonetisation and Amnesties

Demonetisation is taken to result in no exogenous change in white money supply so that only black cash balances are reduced. Consequently, the main effect comes from reduced consumption on account of reduced black wealth. The resulting leftward shift in the IS curve puts downward pressure on interest rates which stimulates black investment and curbs the consumption effect. Consequently, while white income decreases, the effect on black income is uncertain.

In the credit-constrained regime, interest rates remain unaffected so that both white and black incomes contract.

An amnesty is modelled as a one-shot reduction in black wealth and an increase in white wealth. If an amnesty is taken advantage of, it must be the case

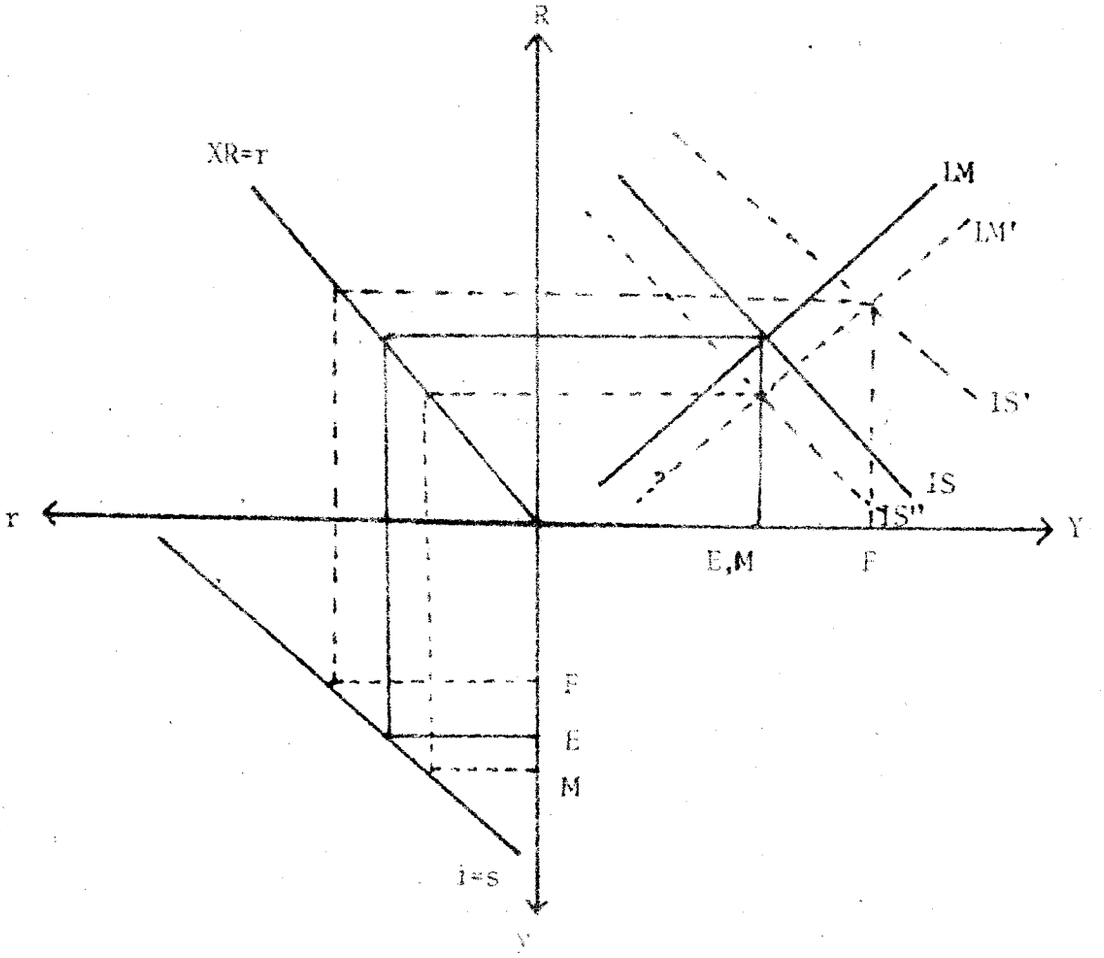

Figure 3. Fiscal and Monetary Policy Effects with Parallel Production

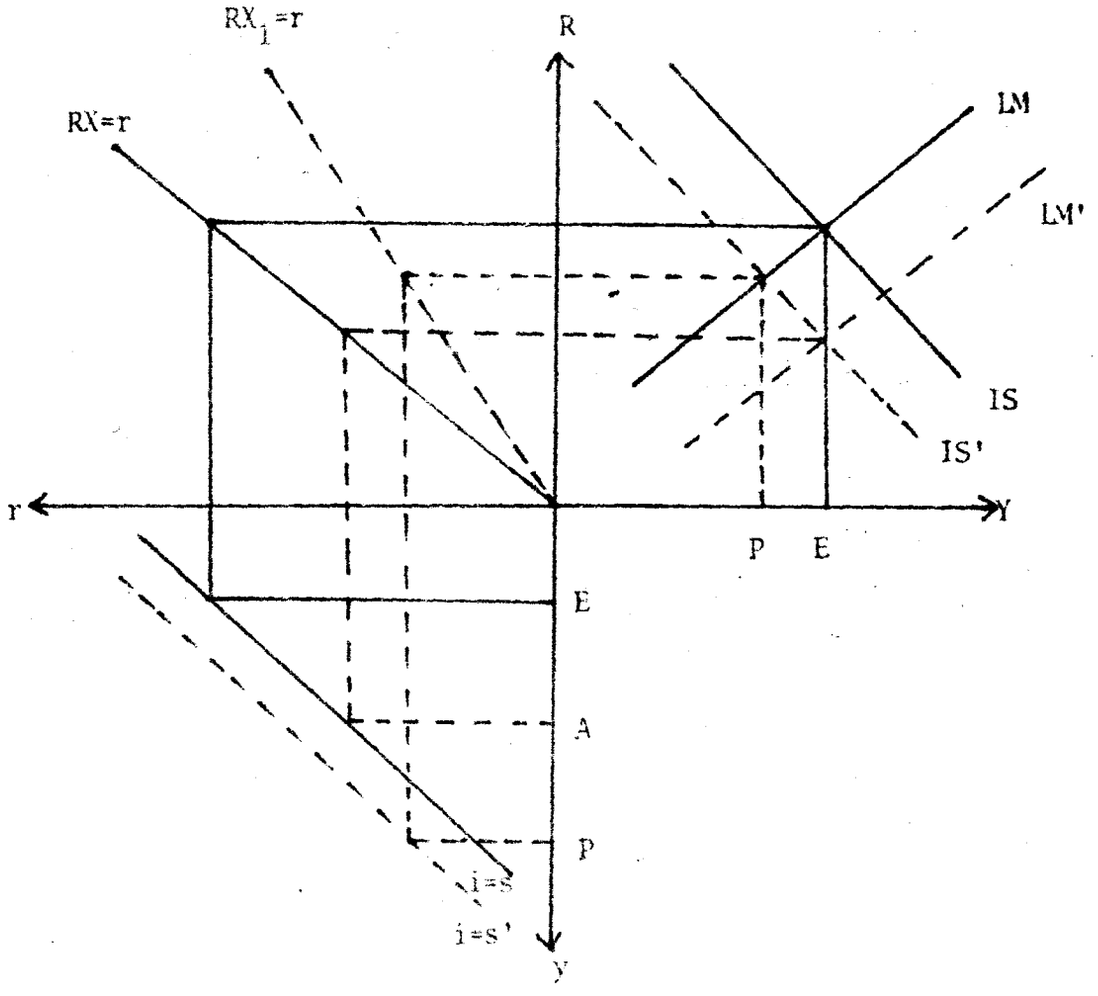

Figure 4. Impact of Stepped up Income Auditing and Amnesty

that the reduction in after-scrutiny black wealth is at least offset by the increase in white wealth. Thus if an amnesty is strictly preferred by an evader it must result in lower initial tax revenues. An initially revenue-neutral amnesty will not change aggregate leakage but will stimulate white saving and black consumption. The expansion in white money supply will lead to declining interest rates which will further stimulate black income generation. Since white money expansion has uncertain effects on white income, black and total income will rise while white income may or may not. Thus, an amnesty has uncertain eventual effects on government revenue but stimulates black activity. Furthermore, it may not lead to an increase in white income. In Figure 3, the new equilibrium with unchanged white income is shown by the line EA. The effects in a credit-constrained regime are qualitatively similar.

Evidently, while demonetisation may be useful in times of excess inflationary pressure, especially in a credit-constrained regime, amnesty has much less to commend it. The various results are summarised in Table 2.

c. Which Policy?

To summarise, all anti-black economy policies involve trade-offs at the least, in terms of white income:

None is effective in curbing the black economy without the possibility of imposing costs on the white sector. Stepped up white income auditing and manesties, in fact, lead to expansion of black activity. It is clear that further work is required to identify useful anti-black economy policies which do not impose costs on the white sector and do not unintentionally stimulate black activity.

6. Conclusions and Extensions

The major findings of the policy analysis in the two preceding sections can be summarised as follows.

In an economy with black production and tax evasion, expansion in government spending has uncertain effects on aggregate income while monetary expansion is stimulative. To restore the efficacy of fiscal expansion, lenient wealth scrutiny may be necessary. However, with respect to white income alone, fiscal expansion has its expected impact while monetary effects are uncertain. Finally, fiscal expansion curbs black production while monetary expansion stimulates it.

With regard to anti-black economy policies, the effects of increased taxes, fines or wealth auditing are uncertain. While increased white income auditing may curb evasion of taxes on white factor incomes, it will stimulate underground income generation. Amnesties stimulate black production and aggregate income generation.

Demonetisation curbs the white economy and will also curb the black economy in a credit constrained regime. Clearly, since amnesties do not curb tax evasion or black income generation and may not stimulate government revenues, they should be eschewed.

The framework, as it stands, is incomplete in at least two major ways. First, laundering markets have not been explicitly incorporated even though the role played by the equilibrium laundering rate has been pointed out. Anecdotal evidence suggests that such markets are crucial for businessmen engaged in illegal production. Secondly, a more sophisticated model of firm behaviour needs to be incorporated into the framework. This extension should permit a better understanding of investment behaviour and the supply side of the economy in the presence of parallel markets. The incorporation of a properly modelled supply side will also permit a distinction to be made between real and nominal variables and allow the study of inflationary implications of parallel markets.

NOTES

1. For a recent survey see Cowell (1985).

2. Specifically, India.

3. Individual subscripts are supressed throughout the section except when aggregating over households in the final section. Throughout the paper, additional subscripts denote partial derivatives with respect to subscript variables. Also, throughout the paper all variables are in real terms. A glossary of notation used is appended for easy reference.

4. For simplicity it is assumed that the cost of detection equals penalties collected, so that there is no net augmentation of government revenues. Also, for simplicity, it is assumed that there are no indirect business taxes.

5. This is true in aggregate. Of course, gross laundering transactions are possible.

6. Local stability conditions and algebraic expressions for multipliers are given in Appendix II. Also, a list of symbols used is given in Appendix III.

7. For a comprehensive survey of both theoretical and empirical work, see Fry (1988).

8. There are some doubts about the validity of this hypothesis based on cross-country evidence though the issue is not yet settled (Fry 1988).

9. We are indebted to Amal Sanyal for helpful discussion on this point.

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APPENDIX I

A Model of Household Behaviour

1.1 Sources and Uses of Funds

We develop here a model of household behaviour over one generation. A household begins with a bequest from its progenitors and atrophies on making a bequest to its descendants, Through its lifetime the household receives income as well as consumes out of the total resources at its command - adding the flow of income to its endowment. The household may also lend in the market for funds. It may choose to participate in the black economy in all its activities. A market is described as a 'black market' when transactions in such a market are sought to be unreported to the State. The rest is carried out in the 'white market'. Consider the sources and uses of funds for such a household.

The household begins with an endowment which may be white (W) or black (w) or both. It also receives income from various activities undertaken. It declares a part of its income and pays taxes on it; a part of its undeclared income could also be detected, in the event of which it has to pay penalties. These two together constitute the household's white disposable income. Total factor income receipts have two components. That part which is reported in firms' books of account is denoted by Y, and that which is not, is denoted by y; the second source clearly constitutes a part of black income. In addition, black income arises due to the undeclared and undetected part of Y.

Black income and wealth have the additional risk being subjected to scrutiny over time. V represents the fraction of black funds remaining after "wealth scrutiny" and P denotes the (constant) probability of escaping detection. Reinganum and Wilde (1985) and Sengupta (1987) show that, when auditing is costly, an audit cut-off policy is superior to the random audit policy implicitly being assumed here. However, they implicitly assume that the probability of audit is equal to the probability of detection and conviction. When this is not the case, the optimal P may not in fact be achievable. In such an event no household would report at the audit cut-off so that random "auditing" and a cut-off policy become equivalent.

The household is able to consume in the proportions it chooses out of white and black income. Consumption out of white and black income are denoted by C and c respectively. White disposable income that is not spent on consumption is white savings (S), which earn pre-tax interest at the rate R and effective rate (after tax and evasion) of R_e. Correspondingly, black disposable income that is not consumed is saved(s), earning interest at the rate r. Households may dissave if they so choose with the constraint that the present value of loans contracted in the black market does not exceed expected undetected interest income. This is a solvency condition imposed on the household's borrowing capacity. Savings and interest earnings are bequeathed as B or b (white or black).

We assume a proportional tax rate on declared income denoted by t. In the event that undeclared income is detected, the penal rate of tax the household pays is ft, $f > 1$. Total government revenues from all sources are denoted by T.

Household income is assumed to be received in small identical pay packets from several sources. Undeclared income from each pay packet is assumed to have an independent chance of being detected. Thus ex-ante expected (after tax and fine) white and black income can be treated as equal to ex-post income with unit probability. This follows from a straightforward application of the binomial probability law given the constant t, f, P and V assumed. Therefore, given undeclared income Y' we have the following expressions for white and black disposable income:

$$Y_d=(Y-Y')(1-t) + Y'(1-P)(1-tf) \qquad (A1')$$

and

$$y_d=VPY'-(1-V)w + vy \qquad (A2')$$

In (A2'), $(1-V)^w$ is the outflow of black wealth due to additional wealth scrutiny operations. The parameter V is determined in a fashion similar to the effective tax rate. Black income inflows and black wealth are held in several parcels each with an independent probability of detection (1-V). If detected, a fraction f',

assumed equal to unity for simplicity, is confiscated. Expected after-scrutiny black funds are, therefore, $V(w+PY'+y)$. As before, expected and ex-post after-scrutiny black funds are taken to be identical.

Households are assumed to maximise the utility function

$$U(C^*)+U'(B^*) \text{ where } C^*=C+c \text{ and } B^*=B+b$$

Given the equality of ex-post and expected values, utility from constant P, f, V and t, a household will either report all its income or none of its income. No income reporting will be strictly preferred if and only if

$$(1-P)\ (1-tf) + VP > (1-t) \qquad (A3)$$

This condition is assumed to hold throughout the paper. We thus conclude that $Y=Y'$. Consequently, white and black disposable incomes are redefined as

$$Y_d=Y(1-P)\ (1-tf) \qquad (A1)$$

$$y_d=VPY - (1-V)w + Vy \qquad (A2)$$

Furthermore, the effective after-tax white interest rate is given under these assumptions, by

$$R_e = R\ (1-P)\ (1-tf) + VRP \qquad (A4)$$

The household white and black budget constraints are

$$(W+Y(1-P)(1-tf)-C)(1+R(1-P)(1-tf)) = B \qquad (A5)$$

and

$$(V(w+y+PY)-c)(1+r)+VRP(W+Y(1-P)(1-tf)-C) = b \qquad (A6)$$

(A5) and (A6) can be combined to yield the total budget constraint

$$(W+Y(1-P)(1-tf)-C)(1+Re)+(V(w+y+PY)-c)(1+r) = B^* \qquad (A7')$$

Note that maximum black consumption is given by

$$c^* = V(w+y+PY)+VRP(W+Y(1-P)(1-tf)/(1+r) \qquad (A8)$$

when both white consumption and black bequests are zero. The term $VRP(W+Y(1-P)(1-tf)/(1+r)$ represents black borrowing capacity. It should be noted that households actually leave pre-tax bequests ($S+W$ and $s+Vw$) for their descendants. However, they derive utility from post-tax bequests, B and b, so that these have to be determined.

Since the number of scrutiny operations can be expected to be an increasing function of time, black funds held for two periods will be less in expected terms than black funds held for one period after penalty

payment. This could be formally accommodated by replacing C_i in (6) and (7) by $V' C_i$, where $1 > V' > V$. This will complicate the algebra and add nothing to insight and is, therefore, eschewed. Instead, we assume that household's consumption expenses are met first out of black funds and borrowing. They call on their white funds to meet consumption expenses only in the event that other sources are exhausted. The obverse of this behaviour is that, to the extent possible, savings are made from white funds. With this assumption we are now in a position to explore the implications of the model.

1.2 The Black Loan Market

If $R_e > r$, it will be optimal for all households to borrow on the black loans market and carry out all saving in the white market. No household will be willing to lend on the black market. The opposite is true if $R_e < r$. Consequently, in equilibrium $R_e = r$ which implies that $r < R$. This discussion ignores the probability of residual saving in the sector with the lower interest rate due to budget constraints.

1.3 Consumption and Saving Functions

Given the argument of the previous subsection we may rewrite (A7') as

$$(N_1 - C^*)(1+r) = B^* \qquad (A7)$$

where $N_1=W+Y(1-P)(1-tf)+V(w+y+PY)$ is the beginning of period household net worth. Under standard assumptions, (A7) and the utility function imply a household consumption demand function of the form $C^*=C^*(R_e,N_1)=C^*(r,N_1)$ with $C^*_r < 0$ and $C^*_N > 0$. The normal restriction on the marginal propensity to consume, $0 < C^*_N < 1$ is assumed. Total savings are given by $Y-C^*-T=S^*$ and total 'leakages' are given by $Y-C^*=T+S^* = L$, where current period government tax collection from the household is

$$T = Y-Y_d-y_d = (1-(1-P)(1-tf)-VP)Y + (1-V)(w+y) \quad (A9)$$

Straightforward calculation yields

$$S^* = \underset{+\ +\ +\ -\ -\ -\ +}{S^*(r,Y,y;W,w,tf,P)} \quad (A10)$$

and

$$L = \underset{+\ +\ +\ -\ -\ +\ -}{L(r,Y,y;W,w,tf,P)} \quad (A11)$$

Formulae for partial derivatives of (A10), (A11) and the consumption function are given in Table A-1. The variable A is an artificial variable which captures the direct effect of an amnesty on the household. An amnesty may be viewed as a one-shot opportunity allowed by the government for households to declare concealed wealth and give up a fraction to the government. (1-a) is the fraction of declared wealth taken by the government. Thus the household gives up black wealth Vw in exchange for white wealth aW. We assume that the amnesty is revenue-neutral so that a=V.

We now turn to the study of black savings. To do this, we take into account the possibility of households being in one of three possible zones. These are illustrated in Figure A1 where N_2, end of period net worth, is plotted against consumption.

These zones may be described as follows:

Zone I: High savers; they save all their white funds and a part of their black funds.

Zone II: The intermediate case; they consume all their black funds making partial use of their borrowing capacity in the black market and save all white funds.

Zone III: High consumers; they consume all their black funds and in doing so exhaust all opportunities of borrowing in the black market and also consume a part of their white funds.

Households in zone III, whose consumption is large relative to total black income and wealth, may be thought of as salary earners. In practice, such households are likely to have both limited black wealth and limited evasion opportunities, though differential evasion opportunities are not modelled explicitly in the formal analysis. These households not only save out of white funds but consume almost entirely out of

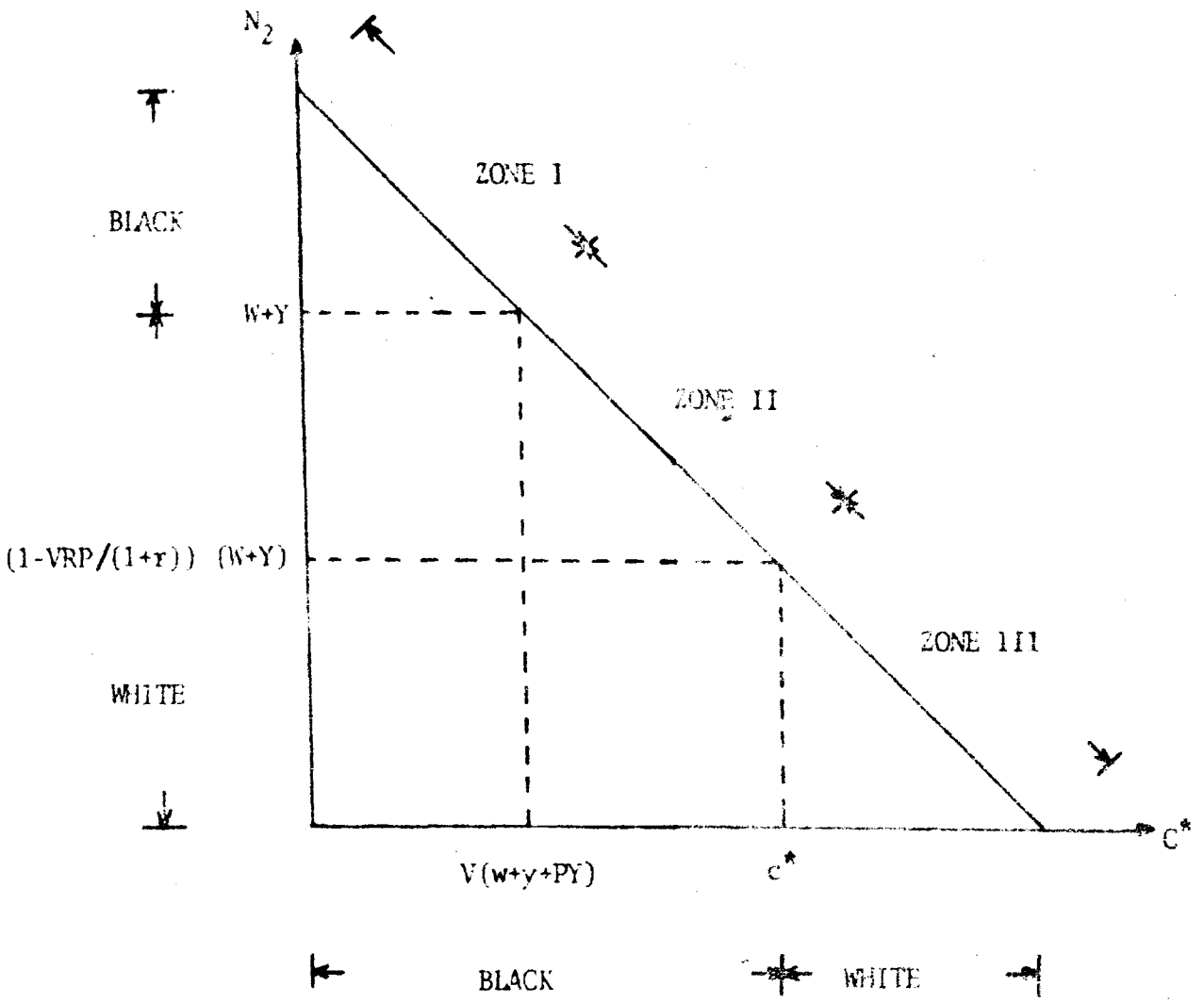

Figure A1. Types of Households

the same funds as well. It is therefore reasonable to hold that their importance to the parallel economy, even when their numbers are large, is likely to be small. For US evidence of this, see Henry (1983) and for Indian evidence see Acharya et al. (1985). Therefore, wherever exogenous effects for zone III households conflict with those for zone I and zone II households, the former effect is assumed to be swamped by those of the latter. This assumption needs to be invoked only in the case of the interest rate effect on black saving. The dependence of black saving demand on different exogenous changes for the three household types is given in Table A2. Finally, since increased Y causes both Y_d and y_d to increase, the effect on black saving is ambiguous for households in all zones. The effect will be small in aggregate and is, therefore, neglected.

These assumptions imply an aggregate black saving function given by

$$s = \underset{+}{s}(\underset{+}{r}, \underset{+}{y}, \underset{-}{W}, \underset{-}{w}, \underset{+}{tf}, \underset{+}{P}, \underset{+}{V}, \underset{-}{A}) \qquad \text{(A12)}$$

Two remarks may be made before leaving this section.

(a) The weak connection between white consumption and white disposable income, which is a consequence of time-dependent effects of wealth scrutiny, appears at first sight to conflict with national accounts figures for most countries. The paradox gets resolved once

it is recognised that consumption expenditure on the white market (from all sources) is what is measured and not white consumption expenditure.

(b) The high marginal propensity to consume out of black income is often cited as one of the damaging effects of parallel economies. Our model shows that these are accompanied by offsetting effects in the white propensity to consume so that the aggregate propensity is relatively unchanged. Thus, this criticism may be misplaced.

TABLE A-1

Effects of Exogenous Changes on Household Consumption and Saving

	C*	S*	L
W	C^*_N	$-C^*_N$	$-C^*_N$
B	VC^*_N	$-VC^*_N$	$-VC^*_N$
Y	XC^*_N	$X(1-C^*_N)$	$1-XC^*_N$
y	VC^*_N	$V(1-C^*_N)$	$1-VC^*_N$
uf	$-YC^*_N(1-P)$	$-(1-P)(Y-YC^*_N)$	$YC^*_N(1-P)$
P	$-YC^*_N(1-uf-V)$	$-(Y-YC^*_N)(1-uf-V)$	$YC^*_N(1-uf-V)$
r	C^*_r	$-C^*_r$	$-C^*_r$
V	$(B+y+PY)C^*_N$	$(B+y+PY)(1-C^*_N)$	$-(B+y+PY)C^*_N$

Note: $X \equiv (1-P)(1-uf)+VP$

TABLE A-2

Effects of Exogenous Changes on Black Saving

Exogenous variable	Household zone			
	I	II	III	All
W	-	-	-	-
w	-	-	-	-
Y	?	?	?	?(S)
y	+	+	0	+
tf	+	+	+	+
P	+	+	+	+
R	+	+	-	?(+)
V	+	+	+	+
A	-	-	0	-

Notes: 1. S : Second order effect.
2. Households cannot move out of their zone, by assumption.

APPENDIX II

Comparative Static Algebra for the Macro Models

Preliminaries: In order to be able to write down algebraic expressions compactly, we define some additional notation in Table A-3.

Alphabetic subscripts denote partial derivatives with respect to subscripted variables unless otherwise defined in Table A-3. In the case of the penal tax rate, tf, the subscript t is used.

The flexible interest rate regime: Differentiating equations (4), (5) and (6) after substituting (3), one gets the total differential system for policy analysis. A necessary condition for local stability is that the system determinant is negative. The system determinant is given by $-n_y$ in Table A-3. Assuming local stability, comparative static multipliers will have the general form $-(\text{numerator})/n_y$, where the numerators are given in Table A-4. The signs of numerators can be directly determined by noting the signs of partial derivatives, as in Table 1, except for three cases. These are as follows:

1. A sufficient condition for $d(Y+y)/dH$ to be positive is $L_Y > L_y$. But $L_Y - L_y = C^*_y - C^*_Y$. From Appendix I this can be found to be $(V-X)C^*_N = (1-P)(V+tf-1)C^*_N$. The last expression is positive whenever evasion is profitable.

TABLE A-3

Additional Notation for Appendix 2

$E_1 = M_R + X i_r < 0$

$E_2 = I_R + X(i_r - L_r) < 0$

$E_3 = i_r - s_r < 0$

$E_4 = i_r - L_r < 0$

$E_5 = M_R(L_y - s_y) - I_R s_y$

$F_j = L_j E_3 - s_j E_4, j = y, w$

$F_j = (L_j - I_j)E_3 - s_j E_4, j = t, p$

$K_j = L_j s_y - s_j L_y$, $j = r, A, w$

$K_j = (L_j - I_j)s_y - s_j L_y$, $j = p, t$

$n_j = M_Y X F_j - (L_Y E_I + I_R M_Y)s_j$, $j = t, p, y, w$

$Y_X = -(i_r E_s + M_R K_r)$

$R_X = M_Y F_y - L_Y s_y i_r$

$y_X = E_3(L_Y M_R + M_Y I_R) > 0$

$Y_H = I_R s_y - X F_y$

$D = s_y(hXE_3 - 1)$ (credit-constrained regime only)

$h = -R_I I_1/(1 - X R_I I_1 s_r) > 0$ (credit-constrained regime only)

TABLE A-4

Numerators of Comparative Static Multipliers for the Flexible Interest Rate Regime

Exogenous change	Comparative static effect on		
	Y	R	y
G increase	$s_y E_1$	$-M_Y s_y$	$-M_Y X E_3$
H increase	Y_H	$L_Y s_y$	$L_Y X E_3$
tf increase	$-E_1 K_t + Y_X X_t$	$M_Y K_t + R_X X_t$	$n_t + y_X X_t$
P decrease	$E_1 K_p - Y_X X_p$	$-M_Y K_p - R_X X_p$	$-n_p - y_X X_p$
Demonetisation	$E_1 K_w$	$-M_Y K_w$	$-n_w$
Amnesty	$-E_1 L_Y s_A + (1-V) Y_H$	$-M_Y L_Y s_A + (1-V) L_Y s_y$	$-s_A (L_Y E_1 + M_Y E_2) + (1-V) L_Y X E_3$

2. In comparative static expressions for the case of demonetisation, the term K_w appears. The sign of K_w may be found by using the household model of Appendix I. From there it can be found that $L_y=1-VC^*_N$, $L_w=VC^*_N$, $s_w=-(1-V)-c_w$ and $s_y=V-c_w$. Now $VC^*_N < c_w$ since the two are equal for households in zones I and II and the former is strictly smaller for households in zone III. Making these substitutions we get $K_w=VC^*_N-c_w-(1-V) < 0$. The multipliers dY/dw and dR/dw may now be seen to be negative as claimed.

3. As discussed in the text, demonetisation is assumed not to lead to any exogenous change in white money supply, while amnesties initially increase white money supply by $(1-V)dA$.

The Credit-constrained regime: Equations (5) and (6') form an independent subsystem of the model determining R and y. To solve the model, first differentiate (6') totally and simplify to get

$$dR = -hd(H-Vw)+hs_y dy+h(s_j dj)^* u \qquad (A13)$$

where $(s_j dj)^*$ is a row vector of differential policy effects on illegal saving and investment, j=tf,P,w,A, and u is a column vector of ones.

Next, totally differentiate (5) and use the resulting total differential equation with (A13) to get various policy multipliers for R and y. These are given in the second and third columns of Table A-5. Using the results of these two columns and the equations R=R(I), i=i(XR), the following equations can be deduced.

$$I=I(H,\ tf,\ P,\ A) \qquad (A14)$$
$$\quad\ -\ \ -\ \ +\ \ -$$

$$i=i(H,\ tf,\ P,\ A) \qquad (A15)$$
$$\quad\ +\ \ +\ \ -\ \ +$$

$$y=y(H,tf,P,w,A) \qquad (A16)$$
$$\quad\ +\ \ ?\ -\ +\ +$$

Substituting (A14) - (A16) into (4') and totally differentiating, allows us to solve for the impact of policy changes on Y. These are given in the first column of Table A-5. In determining the sign of d(Y+y), the term (L_Y-L_y) appears in the multipliers for H and A. As discussed above, these are positive. Table 2 gives the signs of these multipliers.

TABLE A-5

Comparative Static Multipliers for the Credit-Constrained Regime

Exogenous Change	Comparative static effect on		
	Y	R	y
G increase	$1/L_Y$	0	0
H increase	$(I_H+i_H-L_y y_H)/L_Y$	hs_y/D	hXE_3/D
tf increase	$(i_t+I_t-L_t-L_y y_t)/L_Y$	0	$-(s_t+s_r X_t)/s_y$
P decrease	$(-i_p+L_p-I_p+L_y y_p)/L_Y$	0	$(s_p+s_r X_p)/s_y$
Demonetisation	$(L_w+L_y y_w)/L_Y$	0	s_w/s_y
Amnesty	$(I_A+i_A-L_y y_A)/L_Y$	$n(1-V)s_y/D$	$h(1-V)XE_3/D-s_A/s_y$

APPENDIX III

Table of Symbols Used

The paper contains a large number of algebraic symbols. Many of these symbols are used only in a single paragraph and are, therefore,, omitted from this table without inconveniencing the reader. Other symbols are as follows:

1. Used in the model of household behaviour

A	:	Variable measuring the effect of amnesties.
B, b	:	White and black bequests.
C, c	:	Household white and black consumption; $C^* = C+c$.
c^*	:	Upper limit of household black consumption.
f	:	Fine rate on (detected) unreported income.
L	:	Leakages = S+T.
N_1, N_2	:	Beginning and end of period household net worth respectively.
P	:	Probability of escaping detection of unreported income.
R	:	Pre-tax white interest rate.
R_e	:	Effective post-tax white interest rate.
r	:	Black interest rate.
S, s	=	Household white saving and black saving; $S^* = S+s$.
T	:	Tax revenues.
t	:	Income tax rate.
U, U'	:	Utility indices.

V : Fraction of black wealth retained by the household after appropriation by the government due to wealth scrutiny.

W, w : Household white wealth and black wealth respectively.

X : Ratio of the black interest rate to the white interest rate.

Y, y : Gross household beginning of period legal and black factor incomes.

Y' : Undeclared legal income.

Y_d, y_d : White and black disposable income.

2. Additional symbols used in the model of aggregate demand

G : Government expenditure.

H : High powered money stock.

I, i : White and black investment expenditure.

Z : Fraction of white funds realised per unit of black funds laundered.

Note: Subscripts denote partial derivatives with respect to subscripted variables except in the case of Y_d, Y_d and R_e.