On the Macrodynamics of COVID-19 Vaccination

No. 352
31-August-2021
Soumya Datta and C. Saratchand

National Institute of Public Finance and Policy
New Delhi

On the Macrodynamics of COVID-19 Vaccination*

Soumya Datta†
Faculty of Economics,
South Asian University
New Delhi, INDIA
soumya@econ.sau.ac.in

C. Saratchand
Department of Economics,
Satyawati College, University of Delhi
Delhi, INDIA
chandcsarat@gmail.com

August 28, 2021

Abstract

We set up a macroeconomic epidemiological (SIR) model to evaluate the role of vaccination in the interactive dynamics of COVID-19 and the economy. We analytically examine the existence and local stability properties of the four steady states and find strong support for a locally stable interior equilibrium where the economy grows in the continued presence of the pandemic. We also find that it might be possible to attain a pandemic-free economic revival only if the rate of recovery from infection is faster than a threshold level. We examine, both analytically and numerically, the impact of various types of policy interventions, including non-pharmaceutical interventions involving restrictions on economic activities (like lockdowns and travel restrictions) as well as speeding up the rate of vaccination. We find that under reasonable parameter configurations, a combination of large-scale vaccination as well as non-pharmaceutical interventions are required to meet the twin objectives of controlling the pandemic and reviving the economy.

Keywords: COVID-19, health, stability, vaccination, non pharmaceutical intervention .

JEL classification: E10, E61, I10, I18.

---

*An early version of this paper was presented in *2021 Web Conference on Economic Theory and Policy*, organized by National Institute of Public Finance and Policy (New Delhi, India) on March 30-31, 2021. We are grateful to the participants of this conference for their comments and suggestions. This paper builds on the earlier version by incorporating vaccination and temporary immunity for those who recover from COVID-19. The usual disclaimer applies.

†Corresponding author.

1 Introduction

A key characteristic of the COVID-19 pandemic[1], which swamped the whole world in 2020, has been its wave like character of ebb and flow of infections and deaths. These repeated waves of infections and deaths have often elicited policy responses which themselves are not adequately planned and coordinated. In other words, policy interventions such as lock-downs of various types, testing, tracing, vaccinations, economic support measures etc. in many countries have been erratic and short term. This type of unsystematic policy response has partly been because the joint dynamics of the COVID-19 pandemic and the economy have not been adequately understood. This paper attempts to theoretically comprehend, in a preliminary manner, the interactive dynamics of the process involving both the disease and the economy which may underlie this wave like character of not only the pandemic and economic activity but also policy response.

There has been a long tradition of mathematically modeling epidemics going back to Kermack and McKendrick (1927). In these SIR models, the population is divided into three categories: susceptible to infection (S), infected (I), and recovered from infection (R). Subsequent studies extended the framework to include other categories, such as exposed to infection (E), quarantined (Q), leading to further classes of epidemiological models including the SIR, SEIR, SIQR etc. A detailed examination of many these models is contained in Brauer, Castillo-Chavez, and Feng (2019). The swiftness of the COVID-19 pandemic's spread has necessitated a re-look at these models set out for instance in Thomas et al. (2020), Anirudh (2020), Ndaïrou et al. (2020), Ivorra et al. (2020), Kucharski et al. (2020), and Badr et al. (2020). In some of these models, those who are infected acquire lifelong immunity and can never become susceptible in the future. In such models, a process of natural convergence to herd immunity (spontaneous herd immunity) may be possible, though this might come at the cost of a significant loss of lives to the pandemic.

Policy interventions in the context of the COVID-19 pandemic have involved a number of measures, of which we will focus on two; namely, the non-pharmaceutical interventions (in the form of lockdowns, travel restrictions etc.) and vaccination. The non-pharmaceutical interventions reduce the new infections, other things remaining the same. But these measures also create livelihood concerns. If not accompanied with various forms of economic support, the adverse economic impact of these measures result in either political pressure for their withdrawal, or violations of these measures or some combination of both. As a result, policymakers end up responding

[1]COVID-19 is a rapidly transmissible disease, caused by the SARS-Cov-2 virus. The transmission of this disease is often by individuals who are asymptomatic Organization (0009). COVID-19 was first clinically detected in the Wuhan province of China. However it rapidly proliferated to the rest of the world. The World Health Organization publicly declared that COVID-19 was a global pandemic on March 11, 2020 (Organization 0011).

to not only the pandemic but also to the contractionary economic outcome of these non-pharmaceutical policy interventions. As the pandemic proliferated, the diverse types of non-pharmaceutical interventions resulted in significant economic contraction in most countries. This economic contraction had a differential impact on various classes and strata in the population. The economically and socially vulnerable sections often experienced the greatest economic setback (see, for instance, Witteveen 2020; Czymara, Langenkamp, and Cano 2020; Hu 2020; Carlitz and Makhura 2021). The adverse political implications of these consequences, in the absence of significant economic measures, have often resulted in policymakers reversing some of the restrictions on economic activities placed earlier (see, for instance, Tsai et al. 2020; Dagpunar 2020; Leung and Wu 2020; Yu, Qi, and Hu 2020; Rudan 2020). This, one might argue, might have played a key role in re-ignition of the next wave of the pandemic. Pure epidemiological models, which do not take into account the economic impact of these non-pharmaceutical measures, will not be in a position to capture the interactive dynamics of these processes. As a result, a new set of studies have emerged, which attempt to understand the dynamic interaction between the epidemiological and macroeconomic processes.

Toxvaerd (2020) and Makris and Toxvaerd (2020) sought to formally model the response of individuals to the pandemic where physical distancing in society emerged as a consequence of optimal behavior. However, such models do not adequately integrate the role of the macroeconomic considerations in the pandemic. Some early literature on the economic impact of the COVID-19 pandemic dealt with it primarily as a preference or supply shock (cf. Fornaro and Wolf 2020; Castro 2020; Guerrieri et al. 2020). As a result such models did not endogenize the interactive dynamics of the pandemic and economic activity.

Some subsequent studies adopt an alternative approach by trying to combine two models from the fields of epidemiology and New Keynesian macroeconomics, the SIR and the DSGE models respectively, in order to model the joint dynamics of the epidemiological and macroeconomic processes. Eichenbaum, Rebelo, and Trabandt (2020b) represents one of the early examples of this type of modeling. Eichenbaum, Rebelo, and Trabandt (2020b) postulate that representative agents from each of the three categories considered in the SIR model optimize their lifetime utility function (with consumption and leisure as arguments) in an infinite horizon setting. These agents, they argue, incorporate the transition probabilities between these three epidemiological categories under rational expectations. Many of these models assume that there exists perfect information regarding the dynamic trajectories of infection as well as their own current state. The agents may cut back on consumption and labor so as to reduce the probability of getting infected. However, they do not cut back on their consumption and leisure adequately enough due to market failure. This market failure tends to arise from the fact that these agents do not completely internalize the social costs of being infected as their optimization program depend only on the private costs. In this class of models, spontaneous herd immunity is attainable, but it might involve a very high social cost (both in terms of economic losses and public health setbacks). This might be attenuated by putting in place various non-pharmaceutical interventions such as restrictions on economic activities at an initial stage of the pandemic. Eichenbaum, Rebelo, and Trabandt (2020a) is another study that treads a similar path to Eichenbaum, Rebelo, and Trabandt (2020b), but in addition, it also considers alternative market structures.

Many recent studies recognize that both private agents as well as policymakers might possess only imperfect information about the pandemic. Farboodi, Jarosch, and Shimer (2020), for instance, proposes a general equilibrium model of optimization of private agents in an infinite horizon setting. The representative agent derives utility from social activity, but cuta back on social activities in order to try and deal with the pandemic. Farboodi, Jarosch, and Shimer (2020) depart from the assumption of perfect information in that the representative agent does not know whether it is susceptible or infected. Along similar lines, Eichenbaum, Rebelo, and Trabandt (2020c) incorporate the role of testing in a mathematical model where those who are untested do not know their current state, which raises the question of evaluation of the economic costs and benefits of testing in a general equilibrium setting. However, the inadequacy of information, according to Eichenbaum, Rebelo, and Trabandt (2020c), is primarily confined to the current state of health of the agent, which is addressable by testing. Like many other models in this line of literature, the private agents in Eichenbaum, Rebelo, and Trabandt (2020c) know (or are able to learn) the dynamic laws of motion of infection. Therefore they are in a position to undertake the dynamic optimization exercise under rational expectations. The empirical exercise carried out in Eichenbaum et al. (2020) is based on a partial equilibrium model. It seeks to explore the impact of the absence of information in the context of the standard economic theory of risk. Nevertheless, Eichenbaum et al. (2020) postulates that several pieces of information are arguments of the utility function. This includes, for instance, the probability of a susceptible person getting infected or the probability of an infected person dying. This is unrealistic, especially in the early stage of the pandemic. Like many other SIR models, this model does not consider the possibility of reinfection of those who have recovered. Besides, since Eichenbaum et al. (2020) is a partial equilibrium model, it may not be able to adequately capture the joint dynamics of the pandemic spread and related macroeconomic processes.

There have also been studies which do not assume rational expectations on the part of agents in the model. Parui (2021) advances a model of interaction between the economy and the epidemic but does not take into account the role of vaccination. Datta and Saratchand (2021) put forward a model of the interaction between the economy and the pandemic and analyze it without assuming particular functional forms. The current paper seeks to draw on the work in Datta and Saratchand (2021) by incorporating the role of vaccination explicitly.

Vaccination introduces a new dimension into the interactive dynamics of the pandemic and economic activity. Initially, the availability of vaccines is subject to the necessary activity of invention and clinical trials being undertaken. Once vaccines are available, their temporary trajectory might be constrained by the volume of production and the public willingness to get vaccinated. The binding constraint among these two factors might differ among countries. For countries with low levels of vaccination, vaccine hesitancy might not emerge as the binding constraint. In this case, the ability of the government to direct resources to augment vaccine production or acquirement will be decisive. International experience seems to suggest availability of vaccines is a problem mainly with low and lower middle income countries, and generally not a problem with high income countries. If the vaccinated and those recovered from reinfection are amalgamated together for simplicity, then the epidemiological framework will still consist of three categories, namely, the susceptible, infected and 'R', which we can now rechristen as 'resistant'. But it is now fairly evident that immunity either due to vaccination or recovery from infection in the case of COVID-19 is temporary. This is the case because of two factors: firstly, antibodies decline in effectiveness over time; and, secondly, existing vaccines may not be able to offer adequate protection from new variants of SARS-CoV-2.

In the light of the above discussion on the joint dynamics of epidemiological and macroeconomic processes in the course of the COVID-19 pandemic, we identify the following limitations and gaps in the current literature:

1. Many current studies employ some variant of the traditional SIR class of models to represent the epidemiological process. Such models are possibly more suitable for diseases like small pox, where one can acquire lifelong immunity either from vaccination or after successfully recovering from infection. In fact, this acquired immunity is key to achieving convergence in these models to a disease free state through the attainment of spontaneous or vaccine based herd immunity. For COVID-19, as we pointed out earlier, immunity of any kind is temporary.

2. Many (but not all) of the current models of the interactive dynamics of the epidemic and economic activity are based on rational expectations hypothesis, which assumes that the decision makers (either the households or the policymakers) either know, or are able to learn about the law of motion of the infection. However, COVID-19 is a relatively new disease and there is a lack of information about many aspects of this disease. Moreover, a key characteristic feature of this pandemic is the overwhelming presence of asymptomatic patients, who inadvertently transmit the infection. In the absence of universal testing and quarantine, any process of learning the objective model or its parameters would

be quite difficult. Thus, it is quite unlikely that any agent will have access to adequate of information to undertake a dynamic optimization exercise. Their behavior is more likely to depend on some type of heuristics or various rules of thumb.[2]

In the rest of this paper, we set up a model to deal with some (but not all of) these issues. We incorporate the proportion of population who are vaccinated in the category of those resistant from COVID-19 (either due to recovering from the disease or through vaccines). In our framework, all agents operate on the basis of bounded rationality where decisions are based on heuristic considerations. Section 2 sets out the mathematical model and analyzes the existence and local stability of various steady states; section 3 examines the comparative dynamic impact of policy changes on the interior steady state; section 4 numerically examines the local stability and dynamics of the interior steady state; Section 5 concludes the paper by summarizing the key findings and suggesting some areas for future work in this research direction.

2 A Macroeconomic-SIR Model for COVID economy

2.1 Basic Setup

Consider the model of a closed economy before the onset of the pandemic. Let y represent an index of the level of economic activities at time t, $y \in \Re_+$. Let the level of economic activities, in the behavioral or disequilibrium macroeconomics tradition (see, for instance, Flaschel et al. 2015) adjusts in a dynamic multiplier process to the gap between aggregate demand and aggregate supply as follows:

$$\frac{\dot{y}}{y} = \alpha \left[y^d (y) - f(y,i) \right] \quad y^d, f : \Re_+ \to \Re_+,\ f(0) > 0,\ y^d_y < 0, f_y > 0, f_i < 0 \quad (1)$$

where $\alpha > 0$ is the speed of adjustment, while y^d is the aggregate demand, whereas $f(y,i)$ is the aggregate supply. The aggregate supply, from standard production technology, depends directly on the level of economic activities. However, with the onset of the pandemic, the production process gets affected, firstly, due to the workers falling sick, and secondly, due to the imposition of non-pharmaceutical interventions by the government in the form of lockdowns and restrictions on economic activities in order to control the pandemic. Thus, with the onset of the pandemic, the aggregate supply depends inversely on the proportion of population which is infected.[3] If we consider linear functional forms for both y^d and f, with $y^d_y < f_y$, then we have the

[2] A formal discussion of such heuristics in macroeconomic decision making might be found in Grauwe (2012).

[3] In other words, the actual index of economy activity is ceteris paribus lower when there is policy induced restrictions on economic activity. In addition, this will lead to a further decline in aggregate demand, further affecting output negatively.

following:

$$\frac{dy}{dt} = \alpha y \left(1 - \eta y - \kappa i\right) \tag{2}$$

with $\eta, \kappa \geq 0$.

Now let us turn towards the dynamics of the growth of infected population. We borrow this part of the model from the framework of the compartmental SIR models originally introduced by Kermack and McKendrick (1927). Consider, for instance, the simple version of this model found in Brauer, Castillo-Chavez, and Feng (2019, chapter 2.4).The total population, P, population is divided into three compartments, susceptible (S), those infected with COVID-19 (I) and the resistant (R), i.e. those who recover from infection and develop antibodies which provide them with immunity from infection. Therefore, we have

$$S + I + R = P \tag{3}$$

We assume, for simplicity, that the population does not grow or decay during the period under consideration, i.e. the birth and the death rates are equal. Following Brauer, Castillo-Chavez, and Feng (2019, equation 2.10), a proportion βI (where $\beta \geq 0$) of susceptible persons get infected within a period of time by coming into contact with infected persons, whereas a proportion ρ of infected persons recover within the same period and develop antibodies to grow resistant to further infection. We make the following modifications to this simple model:

1. In addition to βSI, another number of susceptible persons, jIy (with $j \geq 0$) become infected by coming together in their workplaces during economic interactions. The spread of infection by this route depends directly on the level of economic activities. This form of spread of the pandemic is fundamentally different from that due to non-economic interaction, as this might take place in workplaces, marketplaces, public transport etc. where large number of strangers meet and it is difficult to isolate the infected persons from others.

2. We introduce vaccination into this model. A fraction $\psi \geq 0$ of susceptible persons directly become resistant without getting infected, by getting themselves vaccinated.

3. However, the antibodies which are produced either from a past infection or vaccines are not life-long. In a finite time vaccinated or recovered persons lose their immunity and become susceptible again. In other words, within any finite period, a fraction $h \geq 0$ of resistant persons become susceptible.

We express this in the form of a modified version of the flowchart of the one found in Brauer, Castillo-Chavez, and Feng (2019, figure 2.1), in figure 1 below. The dynamics of the flow between these three compartments might be expressed in the form

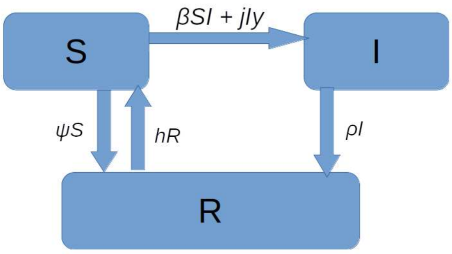


Figure 1: Flowchart for modified macroeconomic SIR model

of the following differential equations:

$$\begin{aligned}\frac{dS}{dt} &= -\beta SI - jIy + \rho I + hR - \psi S \\ \frac{dI}{dt} &= \beta SI + jIy - \rho I \\ \frac{dR}{dt} &= \psi S + \rho I - hR\end{aligned} \tag{4}$$

Defining $s \equiv S/P$, $i \equiv I/P$ and $r \equiv R/P$, we have $s \equiv 1 - i - r$. This makes first of the three equations in (4) redundant. Further, we define $b \equiv \beta P$, so that $b \geq 0$ (since $\beta \geq 0$). Including (2), this allows us to express our model in the form of the following three differential equations:

$$\begin{aligned}\frac{dy}{dt} &= \alpha y \left(1 - \eta y - \kappa i\right) \\ \frac{di}{dt} &= i \left[b \left(1 - i - r\right) - \rho + jy\right] \\ \frac{dr}{dt} &= \psi \left(1 - i - r\right) + \rho i - hr\end{aligned} \tag{5}$$

We should point out here that κ, j and ψ are the three key variables which might be influenced through policy interventions. Before the vaccines arrive, the policymaker might face a trade-off between meeting the public health objective of controlling the pandemic and the economic objective of maintaining economic activities. κ is an indicator of the relative emphasis placed by the policymaker on public health. Higher κ might mean that the policymaker places a greater emphasis on public health objectives vis-à-vis the economic objectives, and is quick to respond to the spread of infections with restrictions on economic activities. j, on the other hand, indicates the effect of economic activities on transmission of the virus. This might largely depend on epidemiological factors (like how infectious the disease is); however, j might be controlled to an extent through policy measures with large-scale use of safety measures including masks, sanitizers, provisions for physical distancing in workplaces etc. Once vaccines become available, however, it allows the country to largely avoid this trade-off by using large-scale vaccination to control the pandemic without curtailing the level of economic activities. However, being able to do so would depend on a number of factors, including policy choices made by the government, political environment in the country (including factors like presence of anti-vaccine sentiment) economic capabilities of the country as well the intricacies of international political economy. η, b, ρ and h, on the other hand, represent macroeconomic and epidemiological processes which, at least in the short-run within the time-period of progress of the pandemic, are largely beyond the control of policy interventions.

2.2 Equilibria and Local Stability

The dynamical system represented by (5) will have *four* steady states, which are listed below:

1. $E_1\left(\bar{y}_1, \bar{i}_1, \bar{r}_1\right) : \left(0, 0, \dfrac{\psi}{h+\psi}\right)$, where the pandemic is fully controlled but at the cost of an economic collapse.
2. $E_2\left(\bar{y}_2, \bar{i}_2, \bar{r}_2\right) : \left(\dfrac{1}{\eta}, 0, \dfrac{\psi}{h+\psi}\right)$, the 'good equilibrium' where the pandemic is brought under control without affecting the economy.
3. $E_3\left(\bar{y}_3, \bar{i}_3, \bar{r}_3\right) : \left(0, \dfrac{bh-\rho h-\psi\rho}{b\left(h+\rho\right)}, \dfrac{\rho\left(b-\rho+\psi\right)}{b\left(h+\rho\right)}\right)$, the 'bad equilibrium' – a failure on all fronts – where there is an economic disaster without bringing the pandemic under control.
4. $E_4\left(\bar{y}_4, \bar{i}_4, \bar{r}_4\right) : \left(-\frac{\kappa bh-bh-\kappa\rho h-\rho b-\kappa\psi\rho}{\kappa hj+\kappa\psi j+\eta bh+\eta\rho b}, \frac{hj+\psi j+\eta bh-\eta\rho h-\eta\psi\rho}{\kappa hj+\kappa\psi j+\eta bh+\eta\rho b}, \frac{\rho j+\kappa\psi j-\psi j+\eta\rho b-\eta\rho^2+\eta\psi\rho}{\kappa hj+\kappa\psi j+\eta bh+\eta\rho b}\right)$, the non-trivial equilibrium, i.e. $E_4 \in \text{int } \Re^3_{+++}$ where both the economic activities and the proportion of infected persons in the economy stabilize at a non-zero level.

We first check whether these steady states are economically and epidemiologically meaningful. For this, we require that all the four steady states lie in the three dimensional unit box, i.e. $(\bar{y}_n, \bar{i}_n, \bar{r}_n) \in [0,1]\times[0,1]\times[0,1]$ for $n = 1, 2, 3, 4$.

We begin by noting that $E_1 \in [0,1]\times[0,1]\times[0,1]$; further, $E_2 \in [0,1]\times[0,1]\times[0,1]$ provided $\eta \geq 1$; in other words both these steady states are economically meaningful with some reasonable restrictions on the parameter values (though economic activities are very unlikely to converge to zero). Next, we note that the 'bad equilibrium', $E_3 \in \Re^3_{+++}$ if and only if

$$\psi \in \left[-\left(b-\rho\right), \frac{h\left(b-\rho\right)}{\rho}\right] \tag{6}$$

We should note that $b > \rho$ is a *necessary* condition for (6) i.e.to be economically meaningful. Further, we note that $\bar{i}_3 \in [0,1]$ always, and the necessary and sufficient condition for $\bar{r}_3 \in [0,1]$ is

$$\psi \leq \frac{bh+\rho^2}{\rho} \tag{7}$$

Combining (6) and (7), the necessary and sufficient condition for E_3 to be economically meaningful is

$$\psi \in \left[-(b-\rho), \min\left(\frac{h(b-\rho)}{\rho}, \frac{bh+\rho^2}{\rho}\right)\right] \tag{8}$$

In other words, a faster vaccination will make the 'bad equilibrium' economically not meaningful. Finally, we turn to the non-trivial equilibrium, E_4. We note that $E_4 \in \Re^3_{+++}$ provided

$$\psi \geq \max\left(\frac{h(b-\rho)}{\rho} - \frac{b(h+\rho)}{\kappa\rho}, -\frac{h[\eta(b-\rho)+j]}{j-\eta\rho}, -\frac{\rho[\eta(b-\rho)+j]}{j(\kappa-1)+\eta\rho}\right) \tag{9}$$

In other words, as long as vaccination rate is sufficiently high, the non-trivial equilibrium is in the real positive orthant. The threshold value of ψ above which E_4 exists in real positive orthant depends on the epidemiological parameters, b, ρ and j as well as the policy parameter κ. We further note that $y_4 < 1$ if $\rho > j$ and $\psi < \frac{\kappa hj+\kappa bh+\eta bh-bh-\kappa\rho h+\eta\rho b-\rho b}{\kappa(\rho-j)}$. Further a sufficient condition for $i_4 < 1$ is that $(\kappa-1)j+\eta\rho > 0$. If $(\kappa-1)j+\eta\rho < 0$ then $i_4 < 1$ if $\psi < \frac{\kappa hj-hj+\eta\rho h+\eta\rho b}{j-\kappa j-\eta\rho}$. If $j < \eta\rho$ then $r_4 < 1$ if $\psi < \frac{(\kappa h-\rho)j+\eta bh+\eta\rho^2}{\eta\rho-j}$. If $j > \eta\rho$ then a sufficient condition for $r_4 < 1$ is that $\kappa > \frac{\rho j-\eta bh-\eta\rho^2}{hj}$.

Next we turn to the local stability analysis of these steady states. We begin by examining the local stability of E_1. The three eigenvalues of the Jacobian matrix to (5) evaluated at E_1 are[4]: α, $-\left[\rho+b\left(\frac{\psi}{h+\psi}+1\right)\right]$ and $-(h+\psi)$. Hence, for $\alpha > 0$, one of the eigenvalues is real and positive; thus E_1 *is locally unstable.*

Next, we turn to the 'good equilibrium', E_2. The three eigenvalues of the Jacobian matrix to (5) evaluated at E_2 are: $\frac{j(h+\psi)+\eta(bh-\rho h-\psi\rho)}{\eta(h+\psi)}$, $-\alpha$ and $-(h+\psi)$. It is obvious that two of the three eigenvalues are always negative; further, if the 'bad equilibrium' $E_3 \in \Re^3_{+++}$ then it follows that one of the eigenvalues is positive and E_2 is unstable. In other words, for the 'good equilibrium' E_2 to be locally stable, it must be the case that the 'bad equilibrium' $E_3 \notin \Re^3_{+++}$ and hence, is economically not meaningful. We further note that E_2 is locally stable if the recovery rate, ρ is sufficiently high, so that the following condition holds true:

$$\rho > \frac{j}{\eta} + \frac{bh}{h+\psi} \tag{10}$$

From inequality (10), we can infer that for the solution to converge to the 'good equilibrium', the recovery rate must be sufficiently high. We further note from the right hand side of (10) that it is more difficult to achieve this inequality if there is an increase in either b or j, representing an increase in the spread of infection through

[4]Calculated using Matlab version R2018a (9.4.0.813654). Codes are available from the authors on request.

either non-economic or economic interactions. On the other hand, an increase in ψ, representing an increase in the speed of vaccination, will make it easier to achieve the 'good equilibrium'.

For determining the local stability properties of the rest of the two steady states, E_3 and E_4, we linearize the dynamical system using a version of Routh-Hurwitz condition found in Flaschel (2009). We compute the Jacobian matrix of the system of differential equations represented by (5) at the steady state E_3 and compute its characteristic equation,

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{11}$$

where λ is the characteristic root and a_1, a_2 and a_3 are the coefficients of the square term and the linear term, and the constant term respectively. For local stability, we require that $a_1 > 0$, $a_3 > 0$ and $a_4 = a_1a_2 - a_3 > 0$ (cf. Flaschel 2009, Theorem A.5, p. 385). We find that at E_3, for the characteristic equation,

$$a_1 = h + \psi + \alpha\left(\kappa\bar{i}_3 - 1\right) \tag{12}$$

and

$$a_3 = \alpha b\bar{i}_3\left(h + \rho\right)\left(\kappa\bar{i}_3 - 1\right) \tag{13}$$

We can rewrite (13) by substituting for $\bar{i}_3$ and rewriting in terms of $\bar{y}_4$ as follows:

$$a_3 = \alpha b\bar{i}_3\left(h + \rho\right)\left(-\bar{y}_4\right) \tag{14}$$

It would be obvious from (14) that if the non-trivial equilibrium, $E_4 \in \text{int } \Re^3_{+++}$, then it follows that at E_3 we will have $a_3 < 0$, hence E_3 is locally unstable whenever E_4 exists and is economically meaningful.

Finally, we turn to the non-trivial equilibrium, E_4. Computing the characteristic equation to the Jacobian evaluated at E_4, we find that

$$a_1 = h + \psi + b\bar{i}_4 + \alpha\eta\bar{y}_4 > 0 \text{ for } \bar{y}_4, \bar{i}_4 > 0 \tag{15}$$

$$a_3 = \alpha\bar{y}_4\bar{i}_4\left(b\eta h + b\eta\rho + hj\kappa + j\kappa\psi\right) > 0 \text{ for } \bar{y}_4, \bar{i}_4 > 0 \tag{16}$$

$$\begin{aligned} a_4 =& \alpha^2 b\eta^2\bar{i}_4\bar{y}_4^2 + \alpha^2\eta^2 h\bar{y}_4^2 + \alpha^2\eta^2\psi\bar{y}_4^2 + j\kappa\alpha^2\eta\bar{i}_4\bar{y}_4^2 + \alpha b^2\eta\bar{i}_4^2\bar{y}_4 + 2\alpha b\eta h\bar{i}_4\bar{y}_4 + \\ & 2\alpha b\eta\bar{i}_4\psi\bar{y}_4 + j\kappa\alpha b\bar{i}_4^2\bar{y}_4 + \alpha\eta h^2\bar{y}_4 + 2\alpha\eta h\psi\bar{y}_4 + \alpha\eta\psi^2\bar{y}_4 + b^2h\bar{i}_4^2 + \rho b^2\bar{i}_4^2 \\ & + bh^2\bar{i}_4 + bh\bar{i}_4\psi + \rho bh\bar{i}_4 + \rho bi\psi > 0 \text{ for } \bar{y}_4, \bar{i}_4 > 0 \end{aligned} \tag{17}$$

In other words, the non-trivial equilibrium, E_4 is locally stable whenever it exists inside the real positive orthant. Hence, we can perform comparative dynamics exercise on it, which we turn to in section 3 below.

3 Comparative Dynamics of Policy Intervention

In this section, we explore the dynamic effect of changes in the parameters on the non-trivial steady state, E_4. In particular, we focus on parameters which have policy implications. There are two possible policy parameters which might be employed by the policymakers in response to the pandemic: the pandemic might be controlled either by (a) bringing in non-pharmaceutical intervention, involving restrictions on economic activities, which will primarily change κ; or (b) increasing the speed of vaccination, affecting ψ. The policy response might also be to improve medical facilities to handle the pandemic, affecting the recovery rate, ρ. However, this is likely to be relevant only at the peak of a wave in the pandemic, and not throughout the pandemic. This is because for most patients except a small percentage of severe cases, COVID-19 does not require sophisticated medical treatment. Hence, the time period required for the majority of patients to recover from COVID-19 is not responsive to policy stance, except at the peak of the pandemic when the medical infrastructure is put under strain by the scale of the pandemic.[5]

We begin by perturbing κ, which has important policy implications. We recall that κ is the sensitivity of the percentage change in the economic activities to the proportion of population infected with COVID-19. An increase in proportion of infected persons in the population might affect economic activities in two ways: firstly, infected persons will temporarily drop out of the workforce, disrupting production; and secondly, increase in proportion of infected persons will induce the policymaker to introduce various non-pharmaceutical interventions involving restrictions on economic activities, especially if the policymaker faces various constraints on the possibilities of quickly increasing the rate of vaccination to control the pandemic. Given that non-pharmaceutical interventions have been extensively used to control the pandemic, κ has emerged as an important policy parameter.

By partially differentiating the non-trivial steady state values of $\bar{y}_4$, with respect to κ, we find the following:

$$\frac{\partial \bar{y}_4}{\partial \kappa} = -\frac{b\ (h+\rho)\ (hj+\psi j+\eta bh-\eta\rho h-\eta\psi\rho)}{(\kappa hj+\kappa\psi j+\eta bh+\eta\rho b)^2} < 0 \text{ for } E_4 \in \Re^3_{+++} \qquad (18)$$

The sign of this derivative is negative since the denominator is positive while the two multiplicative terms in the numerator are both individually positive. The first term $(h+\rho)$ is positive by definition while the second is positive if $\bar{i}_4 > 0$. In other words, non-pharmaceutical interventions involving restrictions on economic activities

[5] A case in point is Italy, a country with second highest index of healthcare in the world (Tandon et al. 2020), had a death rate of nearly 47 percent during the peak of the first wave of the COVID-19 pandemic (March 13, 2020, https://www.worldometers.info/coronavirus/country/italy/) due to the healthcare infrastructure getting completely overwhelmed by the pandemic.

will have a negative impact on economic activities, as we would expect. Next, we partially differentiate $\bar{i}_4$ with respect to κ:

$$\frac{\partial \bar{i}_4}{\partial \kappa} = -\frac{(h+\psi)\, j\, (hj+\psi j+\eta bh-\eta\rho h-\eta\psi\rho)}{(\kappa hj+\kappa\psi j+\eta bh+\eta\rho b)^2} < 0 \text{ for } E_4 \in \Re^3_{+++} \tag{19}$$

The sign of this derivative is negative since the denominator is positive while the two multiplicative terms in the numerator are both individually positive. The first $(h+\psi)$ is positive by definition while the second is positive if $\bar{i}_4 > 0$. In other words, non-pharmaceutical interventions are effective in reducing the steady state percentage of infected persons in the population. Finally, we partially differentiate $\bar{r}_4$ with respect to κ:

$$\frac{\partial \bar{r}_4}{\partial \kappa} = \frac{(\psi-\rho)\, j\, (hj+\psi j+\eta bh-\eta\rho h-\eta\psi\rho)}{(\kappa hj+\kappa\psi j+\eta bh+\eta\rho b)^2} \tag{20}$$

The sign of this derivative depends on whether $\psi > \rho$, since the denominator is positive while the second multiplicative term in the numerator is positive if $\bar{i}_4 > 0$. If $\psi > \rho$, this means that a greater proportion of population is acquiring immunity through vaccination rather than through recovering from the infection. An increase in non-pharmaceutical interventions leave a larger fraction of population without infection, and hence eligible to get vaccinated to acquire immunity. Hence an increase in κ is helpful in immunizing the population quickly. On the other hand, if $\psi < \rho$, then a greater proportion of population is acquiring immunity through recovering from an infection rather than getting vaccinated. In such a scenario, non-pharmaceutical interventions, by preventing infection, slows down this process. Hence, an increase in κ has a negative impact on the proportion of population who are immune or resistant to infection.

Next, we turn to the effect of changing the rate of vaccination, ψ. Partially differentiating $\bar{y}_4$, $\bar{i}_4$ and $\bar{r}_4$ with respect to ψ:

$$\left.\begin{aligned}
\frac{\partial \bar{y}_4}{\partial \psi} &= \frac{\kappa b\,\left(\kappa hj-hj-\rho j+\eta\rho h+\eta\rho^2\right)}{(\kappa hj+\kappa\psi j+\eta bh+\eta\rho b)^2} > 0\\
\frac{\partial \bar{i}_4}{\partial \psi} &= -\frac{\eta b\,\left(\kappa hj-hj-\rho j+\eta\rho h+\eta\rho^2\right)}{(\kappa hj+\kappa\psi j+\eta bh+\eta\rho b)^2} < 0\\
\frac{\partial \bar{r}_4}{\partial \psi} &= \frac{(\kappa j+\eta b)\,\left(\kappa hj-hj-\rho j+\eta\rho h+\eta\rho^2\right)}{(\kappa hj+\kappa\psi j+\eta bh+\eta\rho b)^2} > 0
\end{aligned}\right\} \text{ for } \kappa > \frac{(h+\rho)\,(j-\eta\rho)}{hj} \tag{21}$$

i.e. an increase in the rate of vaccination might increase the level of economic activities, reduce the proportion of infected persons and increase the proportion of resistant or immune persons in the population in the steady state, provided that certain minimum level of non-pharmaceutical interventions are in place which respond to the spread of the pandemic. In other words, for the vaccination to have its beneficial effect, the vaccination efforts need to be complemented with certain amount of non-pharmaceutical interventions as well.[6] κ here plays two types of complementary role:

[6]We should, however, note that if $j < \eta\rho$, then vaccination alone can improve the outcome even without being accompanied by non-pharmaceutical interventions. Given that usually $\eta > 1$,

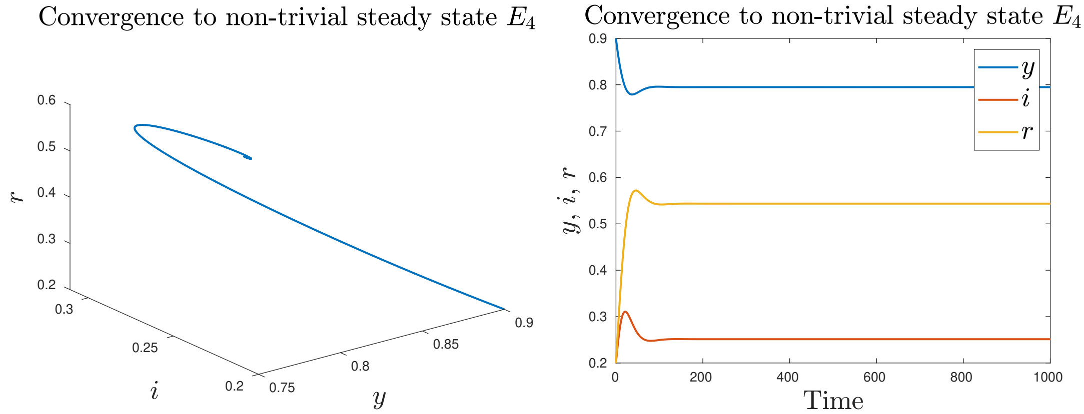
when the proportion of infected in the population increases, it has a depressing impact on y through κ; on the other hand, when the proportion of infected reduces, κ helps revive the economy by reducing its negative impact on y. Overall, this type of policy responsiveness is important to control the pandemic and revive the economy.

4 Numerical Simulation

We next illustrate some of our points made above with numerical examples.[7] Consider the following parameter configuration:

$$\eta = 1.1; \quad b = 0.1; \quad \rho = 0.1; \quad j = 0.1; \quad \psi = 0.01; \quad h = 0.05; \quad \kappa = 0.5$$

The steady states then will be at $E_1 : (0, 0, 0)$, $E_2 : (0.9091, 0, 0.1667)$, $E_3 : (0, -0.0667, 0.0667)$ and $E_4 : (0.7949, 0.2513, 0.5436)$. At the non-trivial equilibrium, around 25.1% of the population is infected with COVID-19. The non-trivial equilibrium is stable for all positive values of α. Starting from an initial point of $(0.9, 0.2, 0.2)$, with $\alpha = 0.1$, we find that the solutions converge to the non-trivial equilibrium. The phase-portrait and the time-series of the solution is shown in figure 2.



(a) Phase diagram (b) Time-aroundseries

Figure 2: Convergence to non-trivial steady state, E_4

As discussed in section 3, the steady state responds to policy shocks. For instance, if the government introduces non-pharmaceutical interventions in order to control the pandemic, so that κ increases from 0.5 to 3, then the solutions converge to

if either j can be brought down through adoption of safety measures (like social distancing, masks, sanitizers and modifications to working conditions in line with COVID-19 protocols) at workplace, or the recovery rate, ρ can be increased, by improving the medical facilities, non-pharmaceutical interventions which restrict economic activities might not be required.

[7] All numerical analyses in this study are performed using Matlab version R2018a (9.4.0.813654). Codes are available from authors on request.

the new non-trivial equilibrium given by $E_4 : (0.5217, 0.1420, 0.3797)$, i.e. the proportion of infected persons in the population is reduced from around 25.1% to 14.2%, but this comes at the cost of a fall in economic activities. However, instead of increasing κ, if the vaccination rate is increased, so that ψ increases from 0.01 to 0.4, then the solutions converge to the non-trivial equilibrium given by $E4 : (0.8974, 0.0256, 0.8718)$, i.e. the rate of infection is reduced to about 2.5% without a large impact on economic activities. However, it might not be possible for every country to reduce the vaccination rate so sharply, at least in the short-run.

The fact that policymakers often have multiple and often conflicting objective of controlling the pandemic, while maintaining the level of economic activities, and are often not in a position (due to economic as well as political reasons) to quickly increase the capacity to vaccinate, might often result in policy flip-flops leading to waves of the pandemic. Consider, for instance, an illustration of simulation under policy shocks shown in figure 3. Initially, the economy is in a pre-pandemic situation with no infected persons. The pandemic breaks out at $t = 500$, leading to sudden rise of infected persons, which also adversely affects the economy with a fall in y. Initial value of κ is at 0.5. The economy quickly settles down to the new equilibrium with lower y and higher i. The pandemic forces the policymaker to respond with non-pharmaceutical interventions, leading to a rise in κ from 0.5 to 3 at $t = 1000$. This leads to a fall in the proportion of infected persons, but this comes at the cost of a sharp fall in economic activities. This economic cost, along with temporary fall in the number of infected persons prompt the government to withdraw some of the non-pharmaceutical interventions at $t = 1500$, leading to the κ falling back to 0.5. This leads to partial revival of the economy, but the cases of infection start rising again, leading to re-imposition of non-pharmaceutical interventions at $t = 2000$. Once again, this leads to fall in cases of infection, but at the cost of a fall in economic activities. Finally, at $t = 2500$, the government is finally able to sharply increase the vaccination rate, leading to a fall in the proportion of infected persons along with negligible impact on the level of economic activities.

The broad experience with many countries might seem to be similar to the pattern outlined above. However, we should note that given the abstractions of our model, the solution discussed above should be taken only as a logical culmination of some of the possibilities and not an actual estimated or calibrated model.

5 Conclusions

The mathematical model set out in this paper is one way of capturing some aspects of the interactive dynamics of the economy and the COVID-19 epidemic with a key role for the dynamics of vaccination. We demonstrate the existence of a steady state with non-zero levels of economic activity, the share of infected in the population

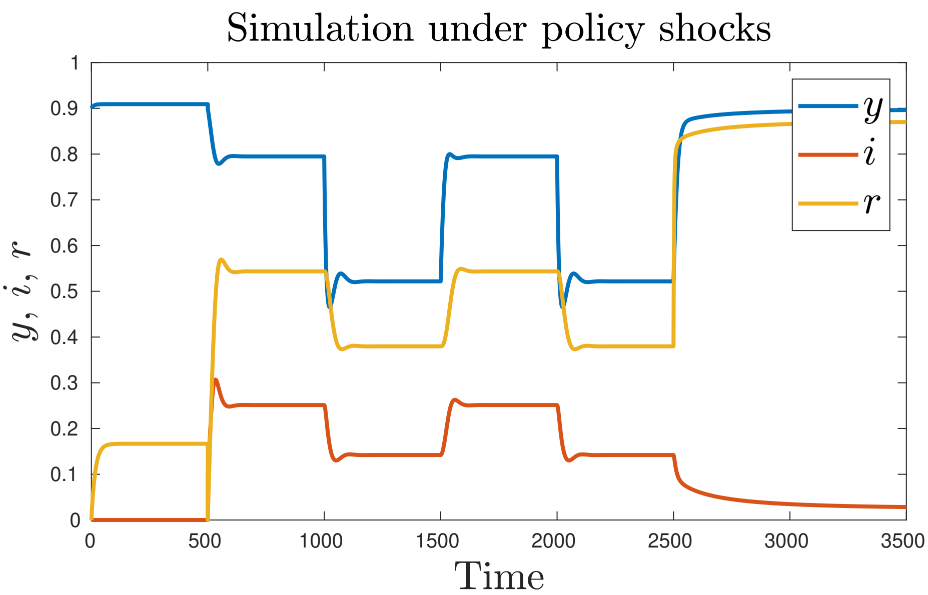


Figure 3: Simulation under policy shock

and the share of recovered (including the vaccinated) in the population. This steady state, if it is economically meaningful (i.e. is in the positive orthant), will be locally stable. On the other hand, a pandemic-free economic recovery might be attainable only when either the recovery rate or the rate of vaccination is above a threshold.

We further find that both the economic as well as the epidemiological variables are sensitive to policy interventions in the form of non-pharmaceutical interventions and increase in the rate of vaccination. If the policymaker more aggressively intervenes in restricting (relaxing) economic activities in response to an increase (decrease) in the proportion of infected persons in the population, it leads to fall in the proportion of infected persons, but this comes at the cost of an economic contraction. The non-pharmaceutical interventions also leads to a rise in resistant population if the rate of vaccination exceeds the rate of recovery, since the non-pharmaceutical interventions allow a larger proportion of population who are susceptible, and hence, available for vaccination. However, if the vaccination rates are lower than the recovery rates, than non-pharmaceutical interventions will lead to a fall in the proportion of resistant population as a smaller fraction of population will acquire immunity through infection.

An increase in the rate of vaccination, on the other hand, might lead to an increase in both economic activity as well as a fall in infection; however this will happen only if the non-pharmaceutical interventions are retained above a threshold level. In other words, the non-pharmaceutical interventions are complementary to vaccination – neither of these two policy responses can replace the other. We also demonstrated, numerically for certain parameter configurations, that policy inconsistencies and flip-flops in the form of periodic imposition and withdrawal of non-pharmaceutical interventions might result in wave-like temporal trajectory of all the three variables. This wave like temporal trajectory can be attenuated only if the rate of vaccination can be increased significantly.

The model of this paper can be extended in a number of ways, of which we mention only three here. Firstly, the recovery rate might positively depend on the proportion of population which is vaccinated (since vaccinated persons, even if infected, might only acquire a milder variety of the disease and recover faster). Secondly, non-pharmaceutical interventions might be endogenized as a separate dynamic process, which generates cyclical behavior endogenously. Thirdly, the fraction of resistant population who loses their immunity might depend on the share of infected persons in the population, because a larger number of infected persons favors mutation of the virus into newer variants which are resistant to existing antibodies and vaccines. We hope to take up some of these issues in future extensions of our work.

References

Anirudh, A. (2020). "Mathematical modeling and the transmission dynamics in predicting the Covid-19 - What next in combating the pandemic". In: *Infectious Disease Modelling* 5, pp. 366 –374.

Badr, Hamada S et al. (2020). "Association between mobility patterns and COVID-19 transmission in the USA: a mathematical modelling study". In: *The Lancet Infectious Diseases.*

Brauer, Fred, Carlos Castillo-Chavez, and Zhilan Feng (2019). *Mathematical models in epidemiology.* Springer.

Carlitz, Ruth D. and Moraka N. Makhura (2021). "Life under lockdown: Illustrating tradeoffs in South Africa's response to COVID-19". In: *World Development* 137, p. 105168.

Castro, Miguel Faria e (2020). *Fiscal Policy During a Pandemic.* Working Papers 2020-006. Federal Reserve Bank of St. Louis.

Czymara, Christian S., Alexander Langenkamp, and Tomás Cano (2020). "Cause for concerns: gender inequality in experiencing the COVID-19 lockdown in Germany". In: *European Societies* 0.0, pp. 1–14.

Dagpunar, John S. (2020). "Sensitivity of UK Covid-19 deaths to the timing of suppression measures and their relaxation". In: *Infectious Disease Modelling* 5, pp. 525 –535.

Datta, Soumya and C Saratchand (2021). "Non-Pharmaceutical Interventions in a Generalized Model of Interactive Dynamics Between COVID-19 and The Economy". In: *Nonlinear Dynamics* 105, pp. 2795–2810.

Eichenbaum, Martin S, Sergio Rebelo, and Mathias Trabandt (2020a). *Epidemics in the Neoclassical and New Keynesian Models.* Working Paper 27430. National Bureau of Economic Research.

Eichenbaum, Martin S, Sergio Rebelo, and Mathias Trabandt (2020b). *The Macroeconomics of Epidemics*. Working Paper 26882. National Bureau of Economic Research.

— (2020c). *The Macroeconomics of Testing and Quarantining*. Working Paper 27104. National Bureau of Economic Research.

Eichenbaum, Martin S et al. (2020). *How do People Respond to Small Probability Events with Large, Negative Consequences?* Working Paper 27988. National Bureau of Economic Research.

Farboodi, Maryam, Gregor Jarosch, and Robert Shimer (2020). *Internal and External Effects of Social Distancing in a Pandemic*. Working Paper 27059. National Bureau of Economic Research.

Flaschel, Peter (2009). *The Macrodynamics of Capitalism: Elements for a Synthesis of Marx, Keynes and Schumpeter*. Second Revised and Enlarged. Berlin Heidelberg: Springer Verlag.

Flaschel, Peter et al. (2015). "A Behavioral Macroeconomic Model of Exchange Rate Fluctuations with Complex Market Expectations Formation". In: *Computational Economics* 45.4, pp. 669–691.

Fornaro, Luca and Martin Wolf (Mar. 2020). *Covid-19 Coronavirus and Macroeconomic Policy*. Working Papers 1168. Barcelona Graduate School of Economics.

Grauwe, Paul de (2012). *Lectures on Behavioral Macroeconomics*. Princeton University Press.

Guerrieri, Veronica et al. (2020). *Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?* Working Paper 26918. National Bureau of Economic Research.

Hu, Yang (2020). "Intersecting ethnic and native–migrant inequalities in the economic impact of the COVID-19 pandemic in the UK". In: *Research in Social Stratification and Mobility* 68, p. 100528.

Ivorra, B. et al. (2020). "Mathematical modeling of the spread of the coronavirus disease 2019 (COVID-19) taking into account the undetected infections. The case of China". In: *Communications in Nonlinear Science and Numerical Simulation* 88, p. 105303.

Kermack, William Ogilvy and Anderson G. McKendrick (1927). "A Contribution to the Math- ematical Theory of Epidemics". In: *Proceedings of the Royal Society of London*. A 115.772, pp. 700–721.

Kucharski, Adam J et al. (2020). "Early dynamics of transmission and control of COVID-19: a mathematical modelling study". In: *The Lancet Infectious Diseases* 20.5, pp. 553 –558.

Leung, Kathy and Joseph T. Wu (2020). "The gradual release exit strategy after lockdown against COVID-19". In: *The Lancet Regional Health – Western Pacific* 1.

Makris, M. and F. Toxvaerd (2020). *Great Expectations: Social Distancing in Anticipation of Pharmaceutical Innovations.* Working Papers in Economics CWPE2097. Cambridge.

Ndaïrou, Faïçal et al. (2020). "Mathematical modeling of COVID-19 transmission dynamics with a case study of Wuhan". In: *Chaos, Solitons & Fractals* 135, p. 109846.

Organization, World Health (11). *WHO Director-General's opening remarks at the media briefing on COVID-19 - 11 March 2020.* Tech. rep.

— (9). *Transmission of SARS-CoV-2: implications for infection prevention precautions.* Tech. rep.

Parui, Pintu (2021). "A simple macro-model of COVID-19 with special reference to India". In: *Metroeconomica* n/a.n/a.

Rudan, Igor (2020). "A cascade of causes that led to the COVID-19 tragedy in Italy and in other European Union countries". In: *Journal of global health* 10.1, pp. 010335–010335.

Tandon, Ajay et al. (2020). "Measuring overall health system performance for 191 countries". In: *Geneva: World Health Organization.*

Thomas, Diana M. et al. (2020). "A Primer on COVID-19 Mathematical Models". In: *Obesity* 28.8, pp. 1375–1377.

Toxvaerd, F. (2020). *Equilibrium Social Distancing.* Working Papers in Economics CWPE2021. Cambridge.

Tsai, Alexander C et al. (Oct. 2020). "Coronavirus Disease 2019 (COVID-19) Transmission in the United States Before Versus After Relaxation of Statewide Social Distancing Measures". In: *Clinical Infectious Diseases.* ciaa1502.

Witteveen, Dirk (2020). "Sociodemographic inequality in exposure to COVID-19-induced economic hardship in the United Kingdom". In: *Research in Social Stratification and Mobility* 69, p. 100551.

Yu, Xinchen, Guoyuan Qi, and Jianbing Hu (2020). "Analysis of second outbreak of COVID-19 after relaxation of control measures in India". In: *Nonlinear Dynamics.*

MORE IN THE SERIES

- Chakraborty, P., (2021). COVID-19 Context and the Fifteenth Finance Commission: Balancing Fiscal Need and Macroeconomic Stability, W.P. No. 351(August).
- Gulati, K., and Anand, T., (2021). Inheritance rights of transgender persons in India, W.P. No. 350 (August).
- Damle, D., and Gulati, K., (2021). Characterising land and property related litigation at the Delhi High Court, W.P. No. 349 (August).

Soumya Datta, is Faculty of Economics, South Asian University New Delhi, INDIA
Email: soumya@econ.sau.ac.in

C. Saratchand, is Department of Economics, Satyawati College, University of Delhi, INDIA
Email: chandcsarat@gmail.com

NIPFP

National Institute of Public Finance and Policy,
18/2, Satsang Vihar Marg, Special Institutional
Area (Near JNU), New Delhi 110067
Tel. No. 26569303, 26569780, 26569784
Fax: 91-11-26852548
www.nipfp.org.in