REVENUE SHARING IN DEVELOPING COUNTRIES: ROLE OF POPULATION IN CRITERIA-BASED TRANSFERS

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ABSTRACT

This paper examines the role of population as a determinant of the overall progressivity of the transfer mechanism and the implications of using dated data relating to population in the revenue dispensation. Data deficiencies and other considerations induce developing countries to use population - shares of States as the dominant determinant of their respective revenue shares. Population criterion by itself is characterised by lack of progressivity and it is not consistent with requirements of vertical equity. If considerable weights are assigned to population, the overall progressivity of the allocative mechanism would be considerably compromised. Further, even in the case of progressive criteria, the use of dated population instead of current year population may result in unintended distortions and penalise the States not only for a more than average population growth rate but also for being poorer. If use of current year population is not feasible, an ex-post evaluation of the losses suffered by different States should be undertaken periodically, and a mechanism for full or partial compensation ought to be instituted.

REVENUE SHARING IN DEVELOPING COUNTRIES: ROLE OF POPULATION IN CRITERIA-BASED TRANSFERS

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I. Introduction

In the allocative mechanism governing the transfer of resources from the Central government to the State governments, the relative sizes of population of different States play a crucial role. Population is used as a scale factor in combination with either equal amount per capita shares for all the States or with per capita shares that vary between States according to per capita incomes or other indicators. In the former case, the relative shares of the States are determined entirely by their relative population sizes. In the latter case also, population would still have considerable importance in determining the overall share of a State. When various criteria are used in combination, the overall impact of the population factor may be considerable. In developing countries, population figures are often used as proxies for other data such as consumption or sales figures, thus increasing the importance of population in the overall revenue sharing mechanism. Further, often there is a considerable lag in the available population figures and the year for which the devolution exercise is done. This may introduce considerable distortions in the revenue sharing mechanism.

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In this paper, an attempt is made to examine the role of population in the devolution mechanism. In particular, attention is focused on two specific aspects, viz., the impact of population as a determining factor on the overall progressivity of the transfer mechanism, and the implications of using dated data relating to population in the allocative exercises. It is shown that the distortion may be such that States are penalised not only for a more than average population growth rate but also for being poorer.

Various bodies concerned with the resource transfer exercise like the Finance Commission and the Planning Commission in India, may be forced to use population data which are old because of the time-lag in the availability of data or as a matter of deliberate choice. For example, in the Indian context, the final year of award of the Tenth Finance Commission was 1999-2000 and it used population data for 1971, i.e. with a time-lag of 25 to 29 years. Since this was made a part of their terms of reference, they had little choice in the matter. Even when this choice was not so externally imposed, the Finance Commissions were using data with considerable time-lag. For example, the last year of award of the Fifth Finance Commission was 1973-74 for which 1961 data were used, implying an information gap of 9 to 13 years. The relative losses to some States and the consequent gains to others may accumulate to substantial amounts, when such procedures are continued over a long period.

For purposes of the analytical discussion in this paper, three criteria are chosen, viz., (i) population criterion, (ii) distance criterion and (iii) inverse-income criterion. Since the basic point made in the paper relates to the distinction between non-progressive and progressive dispensation criteria, the three criteria chosen here, which happen to be used extensively in different federations across the world[1], cover a wide range of situations. In India, a substantial part of the devolution exercise by the Finance Commissions takes place under the aegis of these three criteria. However, recently, the Tenth Finance Commission has dropped the inverse-income criterion. The population criterion is also used by the Planning Commission with a substantial weight. It also uses the distance criterion to some extent.

The outline of the paper is as follows. In section II, relative progressivities of alternative criteria *vis-a-vis* the population criterion are discussed. Section III considers the effects of population changes on allocative shares under different formulae. In section IV, the importance of population in the overall mechanism of devolution consisting of a composite of alternative criteria is considered. In section V, an analytical framework is proposed for considering the implications of the static character of the allocative mechanism due to dated population data. Section VI contains concluding observations and policy implications of the analysis.

II. Population in Alternative Criteria

Indicating per capita income of States by y_i, $i=1,2,\ldots n$, and arranging them in an ascending order $(y_i < y_{i+1})$, with their population indicated by N_i, we may write the shares of States determined under the population criterion as

$$q_i = N_i/\Sigma N_i \qquad (i=1,\ldots.,n) \tag{1}$$

Correspondingly, the per capita shares of the States are given by

$$q_i^* = 1/\Sigma N_i = \phi \text{ (say)}, \tag{2}$$

which is constant for a given distribution of N_i. Thus, under the population criterion, no matter what is the per capita income of a State, it is the same per capita share for all States. The criterion is characterised by a complete lack of progressivity, since,

$$\frac{\partial q_i^*}{\partial y_i} = 0, \text{ for a given } \phi.$$

In the (q_i^*, y_i) space, dispensation under this criterion would be indicated by a horizontal line with an intercept $(=1/\Sigma N_i)$, as indicated in Figure 1. Since, horizontal equity requires equal treatment of equals and inter alia vertical equity calls for unequal treatment of unequals, the population criterion is inconsistent with vertical equity. In this case, $q_i^*=q_j^*$ even though $y_i<y_j$ where i and j take values from 1 to n.

Let Z and Z_i denote standard per capita fiscal capacity and the per capita fiscal capacity of the ith State. Arranging Z_i in a non-descending order, (i.e., $Z_i \leq Z_{i+1}, i = 1,2,....n$), we can write the shares of states under the distance criterion as [2]

$$a_i = \alpha \ (Z - Z_i) \ N_i \qquad (3)$$

Where $\alpha = 1/\Sigma N_i \ (Z - Z_i)$ (4)

Correspondingly, the per capita shares under this criterion are given by

$$a_i^* = \alpha \ (Z - Z_i) \qquad (5)$$

In the (a_i^*, Z_i) space, this may be represented by a straight line with intercept αZ.

In Australia, Canada and Germany, at least a part of revenue transfers is allocated among the States/provinces according to the formulae which have close resemblance with the general distance forumla. The distance is generally identified in terms of fiscal gap or deficiency in taxable capacity. It is represented by the distance of per capita income of a state from the per capita income of the richest state, in India; by the distance of average tax base of the ith province from the standard five-province average tax base, in Canada; and by the distance of fiscal capacity of the ith State from the average fiscal need, in Germany (Srivastava and Aggarwal, 1985). In the subsequent discussion on the distance formula, we work with its Indian version, which is obtained by considering $Z = Y_i$, per capita State Domestic Product (hereafter referred it as `income') that has been used by the Indian Finance Commissions as a proxy for the vector of tax bases and writing $Z = y_n$, the highest per capita income. The Indian formula can be expressed as[3]

$$a_i = \alpha(y_n - y_i)N_i \tag{6}$$

where $\alpha = 1/\Sigma N_i(y_n - y_i)$ (7)

Correspondingly, the per capita shares under this criterion are given by

$$a_i^* = \alpha(y_n - y_i) \tag{8}$$

In the (a_i^*, y_i) space, this may be represented by a straight line with intercept αy_n. Notice that y_n is the highest per capita income among all States, and allocations are made according to the linear distance of y_i from y_n. For a given distribution of (y_i, N_i), α may be treated as a constant, and the slope of the dispensation line is $\partial a_i^*/\partial y_i = -\alpha$, indicating progressivity.

The point of intersection between the (q_i^*) and (a_i^*) lines can be worked out from

$$\alpha(y_n - y_i) = 1/\Sigma N_i$$

which gives

$$y_i = \frac{\Sigma y_i N_i}{\Sigma N_i} = \mu \text{ (say)} \qquad (9)$$

where μ is the (weighted) average per capita income of all States, i.e., the mean income of the country. This implies that compared to the distance criterion, the population criterion would give higher per capita shares to all the richer States with per capita incomes greater than the mean per capita income, and lower per capita shares to all States with per capita income lower than the mean per capita income of the country.

The per capita shares under the inverse-income criterion may be indicated by

$$b_i^* = \beta/y_i, \quad \text{where} \quad \beta = 1/\Sigma(N_i/y_i) \qquad (10)$$

and $$db_i^*/dy_i = -\beta/y_i^2 \qquad (11)$$

indicating progressivity. An extensive discussion of this criterion in comparison to the distance criterion is available in Srivastava and Aggarwal (1993 and 1995). For a given β, this criterion describes a rectangular hyperbola in the (b_i^*, y_i) space as indicated in Figure 2.

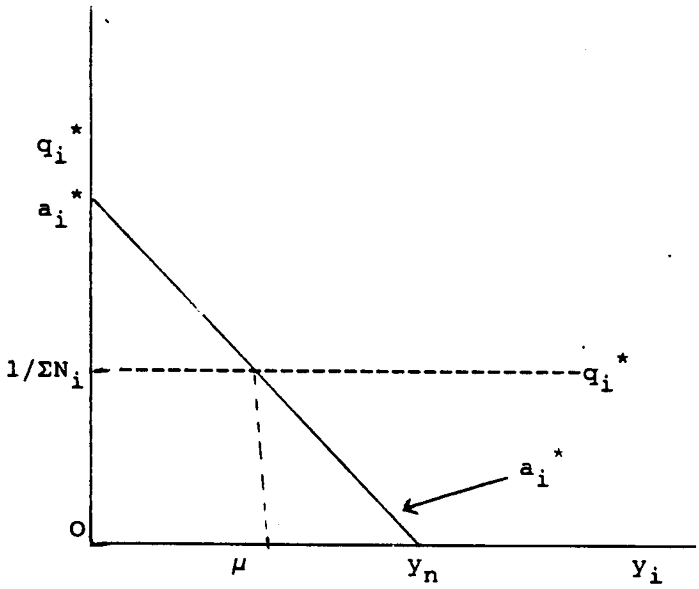

Figure 1

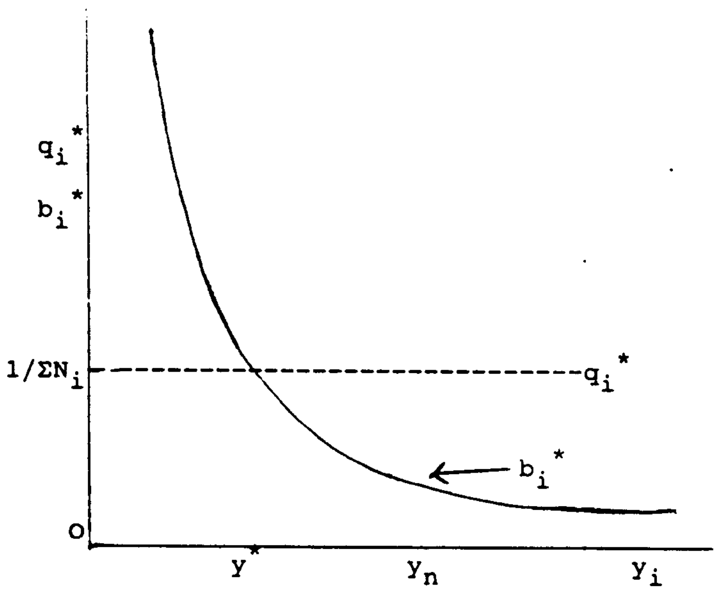

Figure 2

The point of intersection of the q_i^* and b_i^* lines is given by,

$$y_i = \beta \Sigma N_i \quad (= y^*, \text{ say}) \qquad (12)$$

This point would lie to the left of μ, if

$$(\Sigma y_i N_i)\ \Sigma(N_i/y_i) \quad > \quad (\Sigma N_i)^2 \qquad (13)$$

In this case also, the population criterion gives relatively lower shares to States with per capita incomes less than y^*, and relatively higher shares to all States which lie to the right of this point (Figure 2).

To generalise, for any allocative criteria the aggregate and per capita shares can be written as,

$$s_i = \Omega f(.) N_i \text{ and } s_i^* = \Omega f(.)$$

where $\Omega = 1/\Sigma f(.)\ N_i$

where f(.) is a function of arguments such as per capita income. The three criteria considered here are special cases of the above as follows:

$$s_i = q_i \text{ if } f(.) = 1 \text{ with } \Omega = \phi$$

$$s_i = a_i \text{ if } f(.) = y_n - y_i \text{ with } \Omega = \alpha$$

and $s_i = b_i$ if $f(.) = 1/y_i$ with $\Omega = \beta$

In the expression $s_i = s_i^* . N_i$, N_i is entirely a scale factor. Further, relative population of States does not affect the per capita shares[4] of States which are determined entirely by the aggregate population, since in this case $s_i^* = \phi = 1/\Sigma N_i$. In the population formula, therefore, the role of population is entirely that of a scale factor.

In the other two formulae, the relative distribution of population among the States does affect the per capita shares by influencing the terms α and β respectively. In particular, α and β are weighted aggregates of population where the population of poorer States are given higher weights. Thus, we get the result that the relative distribution of population does not affect the per capita shares in the population criteria, but it affects the shares of income-based criteria. In the first case, population is entirely a scaling factor in the determination of State's shares, whereas in the latter case relative distribution of population has a role as a determining factor as well as a scaling factor.

III. Population as a Scale Factor: Some Analytics

The effect of a change in population on per capita shares under the different criteria can be studied in a dynamic perspective also. Here, the responsiveness of the per capita share of the **ith** State, following a change in its population, holding all other things constant, is worked out for different criteria. Thus, under the population criterion,

$$\frac{\partial q_i^*}{\partial N_i} = -\frac{1}{(\Sigma N_i)^2} = -(q_i^*)^2 \qquad (14)$$

A generalised result in this context can be worked out. Defining an allocative criteria based on weighted population shares, where the weights are dependent on a set of State-specific variables, such as per capita income (y_i), we write, the share of a State as

$$s_i = \frac{f(y_i)N_i}{\Sigma f(y_i)N_i} \tag{15}$$

Writing, for a given distribution of (y_i, N_i), $1/\Sigma f(y_i)N_i = \pi$, we have

$$s_i = \pi f(y_i)N_i \tag{16}$$

The corresponding per capita share is then written as

$$s_i^* = \pi f(y_i) \tag{17}$$

The change in s_i^* following a change in N_i, *ceteris paribus*, can then be written as

$$\frac{\partial s_i^*}{\partial N_i} = \frac{\partial \pi}{\partial N_i} f(y_i) + \pi \frac{\partial f(y_i)}{\partial N_i} \tag{18}$$

since,

$$\frac{\partial \pi}{\partial N_i} = \frac{-f(y_i)}{[\Sigma f(y_i)N_i]^2} \quad \text{and} \quad \frac{\partial f(y_i)}{\partial N_i} = o,$$

we may write,

$$\frac{\partial s_i^*}{\partial N_i} = - [\frac{f(y_i)}{\Sigma f(y_i)N_i}]^2 = - s_i^{*2} \tag{19}$$

Thus, for the distance formula,

$$\frac{\partial a_i^*}{\partial N_i} = -(a_i^*)^2 \tag{20}$$

and for the inverse-income criterion,

$$\frac{\partial b_i^*}{\partial N_i} = -(b_i^*)^2 \tag{21}$$

This means that per capita share of the **ith** State whose population increases, would fall in each case. The extent of this fall would depend on the magnitude of the initial share. In comparing the population and the distance criterion, for all States with per capita income less than the mean income of the country (μ), the per capita share would fall relatively more under the distance criterion. Beyond μ, the fall in the share is relatively more for the richer States under the population criterion. Similarly, the critical point of comparison is y^* for corresponding observations in the comparison between the population and the inverse-income criteria.

Defining $e_{Ni}(s_i^*)$ as the elasticity of per capita share of the ith State with respect to a change in its population, we have

$$e_{Ni}(s_i^*) = \frac{\partial s_i^*}{\partial N_i} \cdot \frac{N_i}{s_i^*} = -s_i^{*2} \cdot \frac{N_i}{s_i^*}$$

$$= -s_i^* N_i = -s_i \qquad (22)$$

and,

$$e_{Ni}(s_i) = \frac{\partial s_i}{\partial N_i} \cdot \frac{N_i}{s_i} = N_i(-s_i^{*2}) \cdot \frac{N_i}{s_i} = -s_i \qquad (23)$$

These results are independent of the functional form of s_i (i.e., the functional form of $f(y_i)$ in the allocative criteria) and arise due to the use of population as a scale factor. Specific results for the population, distance and inverse-income criteria are obtained by substituting q_i, a_i and b_i for s_i respectively.

A related property of population responsiveness of allocative shares which is independent of the functional form of $f(y_i)$ in the allocative formula relates to the cross-effects, i.e., the effect of a change in the share of one State following a change in the population of another State. Defining these cross effects as $\partial s_i^*/\partial N_j$, we have,

$$\frac{\partial s_i^*}{\partial N_j} = \frac{Z\partial f(y_i)/\partial N_j - f(y_i)\partial Z/\partial N_j}{Z^2}, \quad \text{where} \quad Z = \Sigma N_i f(y_i)$$

$$= \frac{-f(y_i).\ f(y_j)}{[\Sigma N_i\ f(y_i)]^2}, \quad \text{because} \quad \partial Z/\partial N_j = f(y_j)$$

$$= \frac{-f(y_i).\ f(y_j)}{[\Sigma N_i f(y_i)][\Sigma N_j f(y_j)]} \quad i,j=1,\ldots\ldots n \qquad (24)$$

By symmetry,

$$\frac{\partial s_j^*}{\partial N_i} = -\ s_j^* s_i^* = \frac{\partial s_i^*}{\partial N_j} \qquad (25)$$

This means that a change in the per capita share of the ith State with respect to a unit change in the population of the jth State is the same as the change in the per capita share of the jth State with respect to a unit change in the population of the ith State. The fall in the per capita share of a State with respect to a change in the population of another State is higher, higher is its own per capita share, and the higher is the per capita share of the State whose population changes.

Defining $e_{Nj}(s_i^*)$ as the elasticity of per capita share of the ith State with respect to a change in the per capita population of the jth State, we have

$$e_{Nj}(s_i^*) = \frac{\partial s_i^*}{\partial N_j}\ .\ \frac{N_j}{s_i^*} = -\ s_i^*\ s_j^*\ \frac{N_j}{s_i^*} = -s_j \qquad (26)$$

This means that the percentage fall in the per capita share of ith State following a 1 per cent change in the per capita population of jth State is dependent only on the aggregate share of the jth State. Furthermore, since

$$s_i = s_i^* N_i \text{ and } \partial s_i/\partial N_j = N_i \, \partial s_i^*/\partial N_j$$

we have

$$e_{Nj}(s_i) = \frac{\partial s_i}{\partial N_j} \cdot \frac{N_i}{s_i}$$

$$= N_i(-s_i^* \; s_j^*) \cdot N_j/s_i = -s_j \qquad (27)$$

i.e., elasticity of aggregate share of ith State with respect to a change in the population of the jth State is the same as the corresponding elasticity of the per capita share.

IV. Combination of Criteria: Aggregate Shares and Progressivity

a. Overall Importance of Population as a Determinant

The aggregate share of a State in a dispensation mechanism depends on the relative weights assigned to different criteria which are used in combination. Considering that the set of criteria could be divided into two categories, viz., population based, and progressive criteria like the distance and the inverse-income criteria with population as a scale factor, we can write the aggregate share of a State as

$$A_i = \frac{wN_i}{\Sigma N_i} + (1-w)\left[x \frac{f_1(Y_i)N_i}{\Sigma f_1(Y_i)N_i} + (1-x)\frac{f_2(Y_i)N_i}{\Sigma f_2(Y_i)N_i}\right] \qquad (28)$$

where population, distance and inverse-income criteria are combined with weights, respectively, as w, (1-w)x and (1-w) (1-x), the sum of which is unity. Correspondingly, the aggregate per capita share of the **ith** State is given by

$$A_i^* = w[\frac{1}{\Sigma N_i}] + (1-w) [\frac{xf_1(y_i)}{\Sigma f_1(y_i)N_i} + \frac{(1-x)f_2(y_i)}{\Sigma f_2(y_i)N_i}] \quad (29)$$

It is easily ascertained that population as a determinant of the aggregate or per capita share of a State would have greater importance, the greater is the weight (w) assigned to the population criterion, the smaller is the variance among per capita incomes, and the lower is the degree of progressivity of the progressive criteria.

b. Progressivity of the Combined Criterion

The progressivity of the combined revenue sharing mechanism is given by

$$\frac{dA_i^*}{dy_i} = (1-w) [\frac{xf_1'(y_i)}{\Sigma f_1(y_i)N_i} + \frac{(1-x)f_2'(y_i)}{\Sigma f_2(y_i)N_i}] \quad (30)$$

Treating $\Sigma f(y_i)N_i$ as a constant for a given distribution of (y_i, N_i), we have $dA_i^*/dy_i < 0$, since $f_1'(y_i) < 0$ and $f_2'(y_i) < o$ (for progressive criteria) and $(1-w) > 0$ and $(1-x) > o$. Thus, the combined revenue sharing mechanism results in progressive allocation of dispensation implying that the progressivity of the distance and the inverse-income criteria dominates the regressivity of the population criterion.

In fact, any progressive criteria combined with the population criteria would result in a distributive mechanism which is progressive as a whole. However, the larger is the weight attached to the population criterion, the lower would be the progressivity of the allocative mechanism taken as a whole.

With the combined criterion, the per capita share of the **ith** State can be rewritten as

$$A_i^* = wq_i^* + (1-w)X\,a_i^* + (1-w)\,(1-X)b_i^* \qquad (31)$$

In a static perspective

$$\frac{dA_i^*}{dy_i} = -\left[(1-w)X\,\alpha + (1-w)\,(1-X)\,\frac{\beta}{y_i^2}\right] < 0 \qquad (32)$$

since,

$$\frac{dq_i^*}{dy_i} = 0, \quad \frac{da_i^*}{dy_i} = -\alpha \quad \text{and} \quad \frac{db_i^*}{dy_i} = -\frac{\beta}{y_i^2} \qquad (33)$$

This responds to changes in the weight attached to the population criterion as

$$\frac{d^2A_i^*}{dwdy_i} = \left[X\alpha + (1-X)\,\frac{\beta}{y_i^2}\right] > 0 \qquad (34)$$

i.e., the progressivity of the combined criterion reduces with an increase in w, and the extent of reduction would depend on the relative weights of the distance and the inverse-income criteria and parameters α and β.

V. Dated Population and Current Devolution Shares

Suppose, for the year for which the allocative exercise is being done (current year), the population figures are given by N_i^t. However, instead of using these, some base year figures N_i^o are used. Some of the States would lose in this process while others would gain at their cost. In the ensuing analysis, the implications of this procedure are discussed for the three allocative formulae considered here.

a. Population criterion

Writing q_i^t for shares based on N_i^t and q_i^o for those based on N_i^o, we have,

$$q_i^t - q_i^o = \frac{(\Sigma N_i^o)\ (N_i^t) - (N_i^o)\ (\Sigma N_i^t)}{(\Sigma N_i^o)\ (\Sigma N_i^t)} = L(q_i^o) \text{ say} \qquad (35)$$

Suppose the rate of growth of population of the ith State between periods (o,t) is g_i and the mean rate of growth of population of all States is g, i.e.

$$N_i^t = (1 + g_i)N_i^o$$

and

$$\Sigma N_i^t = (1 + g)\Sigma N_i^o$$

The 'loss' to a State under fixed population shares is then given by

$$L(q_i^o) = \left(\frac{N_i^o}{\Sigma N_i^o}\right)\left(\frac{g_i - g}{1+g}\right) \qquad (36)$$

This difference is positive if $g_i - g > o$. In such a case, States which grow at a rate faster than the average rate of growth of population would be penalised. The loss to a State, satisfying this condition, is higher,

i. the higher is its share in the base year population;

ii. the higher is the difference between its growth rate of population and the average growth rate; and

iii. the lower is the average growth of population, since

$$\frac{dL(q_i^o)}{dg} = - \left(\frac{N_i^o}{\Sigma N_i^o}\right) \left[\frac{1+g_i}{(1+g)^2}\right] \quad \text{for a given } g_i.$$

An illustration is provided with hypothetical data as given in Table 1. In column 1, the base year population figures are given. The growth rates of population and the corresponding current year population figures are given in columns 2 and 3. The growth rate over the period for the aggregate population is 20.86 per cent. The first two States are therefore expected to lose when current population figures are proxied by the base year figures. This is illustrated in the last two columns. Percentage loss or gain when measured in relation to what they should have received as their shares turns out to be substantial in some cases.

Thus, a deliberate policy of penalising a State which shows higher than average growth of population, and correspondingly award those which show a lower than average growth, would be satisfied under the population criterion, characterised by non-progressivity. However, the same policy under progressive dispensations would penalise States not only for their excess population growth, but also for their relatively lower per capita incomes, as shown in the next two sub-sections.

Table 1

Losses with Lagged Data Under the Population Criterion: An Illustration

N_i^o	g_i (%)	N_i^t	q_i^o (%)	q_i^t (%)	$(q_i^o - q_i^t)$ (% points)	$\frac{q_i^o - q_i^t}{q_i^t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
100	30.00	130	28.57	30.73	-2.16	-7.03
80	25.00	100	22.86	23.64	-0.78	-3.29
50	20.00	60	14.29	14.18	0.11	0.78
30	20.00	36	8.57	8.51	0.06	0.71
40	12.50	45	11.43	10.64	0.79	7.42
50	4.00	52	14.29	12.29	2.00	16.27
350	20.86	423	100.00	100.00	0.00	0.00

b. Distance criterion

The share of a State, based on current population data under the distance formula would be

$$a_i^t = \frac{(y_n - y_i)N_i^t}{\Sigma(y_n - y_i)N_i^t} \qquad i=1,\ldots\ldots,n \qquad (37)$$

The corresponding shares for the same year based on fixed base-year population data would be

$$a_i^o = \frac{(y_n - y_i)N_i^o}{\Sigma(y_n - y_i)N_i^o} \qquad (38)$$

Both sets of shares refer to the same (current) year per capita income, y_i.

Writing, $\Sigma(y_n - y_i)N_i^t = x_t$

and $\Sigma(y_n - y_i)N_i^o = x_o$

we have,

$$a_i^t - a_i^o = \frac{1}{x_t x_o} \left[(y_n - y_i) \{x_o N_i^t - x_t N_i^o\} \right] \qquad (39)$$

Hence, $a_i^t - a_i^o > o$,

according as

$$x_o N_i^t > x_t N_i^o \qquad (40)$$

Assuming, the rate of growth of population between period t and o, is given by g_i for the ith State,

we have $N_i^o (1+g_i) = N_i^t$

Hence, condition (37) could be written as

$$g_i > \frac{x_t - x_o}{x_o} \tag{41}$$

The value of the expression on the right hand side can be worked out to be:

$$\frac{x_t - x_o}{x_o} = \frac{\Sigma(y_n - y_i) g_i N_i^o}{\Sigma(y_n - y_i) N_i^o} = g^* \tag{42}$$

Thus, the share of a State based on fixed population shares would be less than that based on current population shares, if

$$g_i > g^* \tag{43}$$

g^* is the critical value of g_i which determines whether a State would lose or gain in relation to a situation where only current population figures are used. Notice that g^* is different from g, which is the average rate of growth of population.

The loss to the State, with population growth satisfying this condition, is given by

$$a_i^t - a_i^o = \frac{1}{x_t} [(y_n - y_i) N_i^o \{g_i - \frac{x_t - x_o}{x_o}\}]$$

$$a_i^t - a_i^o = \frac{1}{x_t} [(y_n - y_i) N_i^o \{g_i - g^*\}] \tag{44}$$

Thus, a State which shows a population growth which is greater than g^* (which is different from the mean growth rate) would have a lower share. In such a case, the loss would be higher for a State, higher the difference between its per capita income from the highest per capita State ($y_n - y_i$), higher the population in the base year (N_i^o) and the larger the difference between its population growth between periods (o and t) from that of weighted average of population growth in all the States (g^*), where the weights are linear income distances. On the other hand, a State which shows a population growth which is lower than g^* would have a higher share. In such a case, the gain would be higher for a State, higher the difference between its per capita income from the highest per capita ($y_n - y_i$), higher the population in the base year (N_i^o) and the larger the difference between its population growth between periods (o and t) from that of g^*.

These observations imply that among the States which satisfy the condition $g_i > g^*$, poorer and larger States will be penalised more (higher $y_n - y_i$ and N_i^o), and among the States which satisfy the condition of $g_i < g^*$, richer and smaller States will gain relatively less.

It is important to note that the use of the progressive criterion implies a shift in the critical value of population growth rate from g (as in the population criterion) to g^*. If $g > g^*$, some States may lose even if their population grows at a rate which is less than the mean rate of growth of population. Correspondingly, if $g^* > g$, some States gain even if their population grows at a rate faster than the mean rate.

To illustrate some of the points raised in this section, a hypothetical table using the same population figures as in Table 1 is constructed by using the distance criterion (see Table 2). The larger States are the relatively poorer States in this example. These States also show larger rates of growth of population. Using base year population figures as compared to the current year population figures indicates that the loss is concentrated for the largest and poorest State while all other States including some in which population is growing at more than the mean rate, gain at its cost. From a comparison of Tables 1 and 2, it would be noted that the second State also gains under the distance criterion in contrast to a loss under the population criterion even though its population increases by more than the average rate (g_i=25 per cent). This is because $g^* = 25.55 > g = 20.86$. From Table 2, it may also be noted that between the States with the same growth rate of population the gain is higher for the State with higher initial population and higher per capita income gap $(y_n - y_i)$ (see rows 3 and 4). These relative losses or gains, may involve substantial sums when they are accumulated year after year over a long period of time.

A general condition for comparing g and g^* can be worked out as below.

$$g > g^*$$

$$\text{if} \quad \frac{\Sigma g_i N_i^o}{\Sigma N_i^o} > \frac{\Sigma (y_n - y_i) g_i N_i^o}{\Sigma (y_n - y_i) N_i^o} \tag{45}$$

$$\text{or if } (\Sigma g_i N_i^o)\,[\Sigma (y_n - y_i) N_i^o] > [\Sigma (y_n - y_i) g_i N_i^o]\,[\Sigma N_i^o]$$

$$\text{or if } y_n [\Sigma g_i N_i^o]\,\Sigma N_i^o - (\Sigma g_i N_i^o)\,\Sigma y_i N_i^o > (\Sigma N_i^o) y_n \Sigma g_i N_i^o - (\Sigma N_i^o)(\Sigma g_i y_i N_i^o)$$

$$\text{or if } (\Sigma N_i^o)\,\Sigma g_i y_i N_i^o > (\Sigma g_i N_i^o)\,(\Sigma y_i N_i^o)$$

Table 2

Effect of Using Fixed Population Shares Under the Distance Criterion: An Example

N_i^o	g_i	Y_i	y_n-y_i	$(y_n-y_i)N_i^o$	$(y_n-y_i)N_i^t$	a_i^o (%)	a_i^t (%)	$a_i^o-a_i^t$ (% points)	$(a_i^o-a_i^t)/a_i^t$ (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
100	30.0	10	90	9000	11700	43.06	44.59	-1.53	-3.43
80	25.0	20	80	6400	8000	30.62	30.49	0.13	0.43
50	20.0	30	70	3500	4200	16.75	16.01	0.74	4.62
30	20.0	60	40	1200	1440	5.74	5.49	0.25	4.55
40	12.5	80	20	800	900	3.83	3.43	0.40	11.66
50	4.00	100	0	0	0	0	0	-	-
				20900	26240	100.00	100.00	0.00	0.00

or if $$\frac{\Sigma g_i y_i N_i^o}{\Sigma g_i N_i^o} > \frac{\Sigma y_i N_i^o}{\Sigma N_i^o} = \frac{\Sigma g y_i N_i^o}{\Sigma g N_i^o}$$

or if $$\Sigma g_i y_i N_i^o > \Sigma g y_i N_i^o, \text{ as } \Sigma g_i N_i^o = \Sigma g N_i^o \qquad (46)$$

If all g_i are equal (=g, say), then, two sides would be equal. The terms $(y_i N_i)$ relate to aggregate incomes, i.e., per capita income multiplied by population. As such the condition derived above indicates that if States having a higher share in aggregate income (considered with reference to base year population) experience a rate of growth of population (g_i) which is more than the mean growth rate of population g, then the critical value of rate of population growth (g^*) would be less than the mean growth rate g. In this context, it may be noted that a State with lower per capita income can have a higher share in aggregate income because of a higher population share.

c. Inverse-income criterion

In the case of the inverse-income formula, as in the case of distance formula, the difference between the shares with current and fixed year population, i.e., between b_i^t and b_i^o can be worked out. Writing,

$$W_t = \Sigma(N_i^t/y_i) \quad \text{and} \quad W_o = \Sigma(N_i^o/y_i),$$

We have,

$$b_i^t - b_i^o = \frac{1}{W_o W_t} \left[\frac{N_i^t W_o}{y_i} - \frac{N_i^o W_t}{y_i} \right]$$

$$= \frac{N_i^o}{y_i W_o W_t} \left[g_i \, \Sigma N_i^o/y_i - \Sigma g_i N_i^o/y_i \right]$$

$$= \frac{N_i^o}{y_i W_t} [g_i - g^{**}] \quad (47)$$

where $$g^{**} = \frac{\Sigma g_i N_i^o / y_i}{\Sigma N_i^o / y_i} \quad (48)$$

Thus, a State with population growth greater than g^{**} (which is different from the mean growth rate) would have a lower share. In such a case, the loss would be higher for a State,

i. the poorer the State (greater value of $1/y_i$);

ii. the larger the initial size of population (N_i^o) of the State; and

iii. the larger the difference between its population growth and g^{**}.

On the other hand, a State with population growth lower than g^{**} would have a higher share. In such a case, the gain would be higher for a State, the poorer the State, the larger the State and the larger the difference between its population growth and g^{**}.

These observations imply that among the States which satisfy the condition $g_i>g^{**}$, poorer and larger States will be penalised more, and among the States which satisfy the condition of $g_i<g^{**}$, ceteris paribus richer and smaller States will gain relatively less. These points relating the inverse-income criterion are illustrated in Table 3 which uses the same population and income data sets as in Tables 1 and 2. In this case, g^{**} works out to be 26.27. It may be noted that between the States with same growth rate of population, the relative gain of the richer and smaller State is lower (rows 3 and 4). Further, it may be noted that in some cases the effect of lower rate of growth of population dominates the effect of higher per capita income and lower initial size of the State (rows 2 and 3) and in some cases the effect of substantially higher per capita income

Table 3

Effect of Using Fixed Population Shares:
Inverse Income Criteria

N_i^o	Y_i	g_i	N_i^o/y_i	N_i^t/y_i	b_i^o (%)	b_i^t (%)	$b_i^o-b_i^t$ (% points)	$b_i^o-b_i^t/b_i^t$ (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
100	10	30.0	10.00	13.00	58.24	59.96	-1.72	-2.87
80	20	25.0	4.00	5.00	23.30	23.06	0.24	1.04
50	30	20.0	1.67	2.00	9.73	9.23	0.50	5.42
30	60	20.0	0.50	0.60	2.91	2.77	0.14	5.05
40	80	12.5	90.50	0.56	2.91	2.58	0.33	12.79
50	100	4.0	0.50	0.52	2.91	2.40	0.51	21.80
350			17.17	21.68	100.00	100.00	0.00	0.00

and substantially lower initial size of the State dominates the effect of lower rate of growth (rows 2 and 4 and rows 3 and 5). The relative gain to higher income States in the case of the inverse-income criterion is substantial.

In other words, use of static population ratios, when progressive criteria are being used, would be biased against poorer and larger States if their population growth rate exceeds g^* and g^{**}, and favourable to these States if their population growth rate falls short of g^* and g^{**}, respectively with reference to the distance and inverse-income criteria.

VI. Concluding Remarks

It has been argued in this paper that in devolution mechanisms where considerable weights are assigned to population as a determining factor, the overall progressivity of the allocative mechanism is considerably compromised. Furthermore, if dated population is used over a long period, substantial and unintended distortions creep into the system. Using fixed base year population shares rather than current population shares means that as far as the population criterion is concerned losses would be larger, the larger is the size of the State and the larger is the excess of the population growth rate of a State relative to the average growth rate of population. When progressive dispensation criteria are used, this same procedure of using static population shares would not only be a penalty for showing more than average population growth but also for being low on the income scale, i.e., the poorer the State, the higher would be the implied loss. The relevant results for different allocation criteria are given in Table 4. Most of the losses are likely to be concentrated in the group of States which are poorest and largest. To the extent that the devolution criteria are designed as a compensatory mechanism, penalising States for being poor and large are perverse features of such a mechanism.

This analysis has direct relevance for the Indian resource transfer mechanism where this mechanism appears to have been perverted by insisting that 1971 population data should be used in allocations as far away as the year 2000 A.D. Such a policy does little to control population but rewards handsomely richer States either unintentionally or even by design. The accumulated losses and gains could add to substantial sums. The appropriate procedure is to use current year population shares. Minimum distortions would occur if officially projected population figures are used for the relevant years of award. If this is not feasible, an ex-post evaluation of the losses suffered by different States should be undertaken periodically, and a mechanism for full or partial compensation ought to be instituted.

Table 4

Losses to States with Faster Growth of Population Due to Use of Dated Population in the Allocation Criteria

Criterion	Formula for determining loss to the ith State
Population	$(N_i^o/\Sigma N_i^o)\ [(g_i-g)/(1+g)]$ where $N_i^t = N_i^o(1+g_i)$ and $\Sigma N_i^t = (\Sigma N_i^o)(1+g)$
Distance	$(1/x_t)\ [(y_n-y_i)N_i^o(g_i-g^*)]$ where $g^* = (x_t-x_o)/x_o$, $x_t = \Sigma(y_n-y_i)N_i^t$, and $x_o = \Sigma(y_n-y_i)\ N_i^o$
Inverse-income	$\frac{N_i^o}{y_i W_t}(g_i - g^{**})$ where $g^{**} = \frac{\Sigma g_i N_i^o/y_i}{\Sigma N_i^o/y_i}$ and $W_t = \Sigma(N_i^t/y_i)$

Notes:

1. N_i^o and N_i^t denote population of the ith state in the base and current periods respectively.

2. g denotes average growth rate of population of all States and g_i denotes growth rate of population of the ith State.

3. y_i and y_n denote respectively per capita incomes of the ith State and of the highest per capita income State.

NOTES

1. Alternative versions of both the distance and the inverse-income criteria are used in different federations. The similarities and differences in the alternative versions used in different federations are discussed in Srivastava and Aggarwal (1995).

2. For details see Srivastava and Aggarwal (1993 or 1995).

3. This is subject to one further modification, namely the distance $(y_n - Y_i)$ for the highest income state, which would be zero, is taken as $(y_n - y_{n-1})$ and correpondingly, the denominator is also adjusted.

4. Population can affect per capita shares if it is used in a non-linear way. Consider for example, an allocative criterion based entirely on population as

$$q_i = \frac{N_i^k}{\Sigma N_i^k} \qquad k \neq 1$$

This could be written as

$$q_i = \frac{(N_i^{k-1}) N_i}{\Sigma N_i^k}$$

where the per capita share is given by

$$q_i^* = (N_i^{k-1} / \Sigma N_i^k)$$

In this case the use of population would have a role more than that of just a scaling factor.

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