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A HYBRID MODEL OF GROWTH WITH OVERLAPPING GENERATIONS

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Introduction

One of the major issues concerning the incidence of tax is whether it discourages or encourages the growth of economy. However, most of the authors have confined their analysis to the steady state framework, in which the rate of growth of all relevant magnitudes remains constant over time (for a non steady state approach see Boadway (1977)).

In a one commodity-one class Solow type model the condition for steady state is:

$$\dot{k} = sf(k) - nk$$

where k is capital-labour ratio, s is the marginal propensity to save and n is the exogenously given population growth rate. Since, the growth rate of the economy is fixed by the population growth rate n, the impact of a tax on the long run growth rate can not be studied.

Atkinson and Stiglitz (1980) proved that a tax on capital can influence the dynamic behaviour of the economy via the savings function under the assumption that s is function of k and the tax rate. If savings is a negative function of the tax rate, then, after the imposition of tax there will be a lower k, which ensures the steady state property of the model. The fall in k leads to a rise in the gross return to capital and a fall in the wage rate. However, economy is growing as usual at the rate n with a lower capital:labour ratio and a lower per capita income.

Now if we replace the classical savings function by life-cycle savings in a 'overlapping-generations' model as developed by Diamond (1965), then any tax on capital to make a transfer to the older generations (retired workers) will lead to a fall in the value of k, but if the revenue is distributed among the younger generations (current workers), k will rise because younger generations is the only source of savings. Again, the interesting observation is that the long run growth rate is unchanged.

There are other works, e.g. Jha (1987), Grieson (1975) and Boskin (1978) which deal with several important aspects of tax policies in a growing economy.

However, a neoclassical growth model is not suitable for the exercise mentioned above due to many reasons. In a Solow type growth model long run growth rate (g) is fixed by the population growth rate (n). Therefore, the causality runs from growth to distribution. A fiscal policy can affect the growth rate by its impact on the distribution of total income. A fixed coefficient neoclassical growth model [see Marglin (1984)] which is closed by the closure g=n, is also not suitable for this purpose for various reasons Most important among them is that the model is unstable. Even if we overcome the stability problem by converting it into a two class model (workers and profit earners with different savings propensities) which ensures a positive elasticity between savings and the rate of profit, the very existence of the closure g=n would not permit us to study the impact of a fiscal policy on the long run growth rate.

Therefore, our objective is to develop a HYBRID growth model, which is not closed by the assumption $g=n$ and the casualty runs from distribution to growth. The model is called hybrid because it makes use of both the neoclassical utility maximization technique and a typical neo-Marxian closure, namely $W=W^*$, where W is the wage rate.

These two elements of two distinctly different growth models will ensure three things in our model. These are as follows:

1. Possibility of divergence between population growth rate (n) and the rate of growth of output (g) (i.e., possibility of unemployment).

2. Causality runs from distribution to growth.

3. Unlike neo-Marxists, given the parameters of the model, an individual is free to maximize his life time utility.

The Model

In our model we assume each individual lives for two periods, a year of work and a year of retirement. Consumption takes place after the harvest A working individual consumes out of his own assets (i.e, wage income which he receives from the retired groups) and saves for his retired consumption, we also assume overlapping generations and no bequests.

It is assumed that all individuals are identical in preferences and each allocates his wage between consumption in the two periods of his economic life (C^1 and C^2) according to maximization of a "Life-Cycle" utility function of the form $U(C^1, C^2)$ subject to the life-cycle budget constraint,

$$C^1 + \frac{C^2}{1+r} = W$$

Where r is the rate of profit and W is the wage rate.

Production in our model is characterized by a one-commodity model in which corn is produced by means of a fixed co-efficient technology in which the only inputs are seed corn and labour. Output is growing at a constant rate g. The current output level is X_t. The production relationship is thus

$$X_t = C_t + a_1 X_t (1+g) \dots\dots\dots\dots\dots\dots (1)$$

C_t is total consumption of retired and younger generations from this year's harvest and a_1 the requirements of seed corn per unit of corn harvest. The only other technological parameter is a_0, which represents the labour requirements per bushel of corn output. We further assume that total labour requirements to produce X_{t+1} is one, in other words, $a_0 X_t (1+g) = 1$ (This assumption is made to remove one extra unknown from our model),

Therefore

$$1 = a_1 (1+g) + C_t / X_t \dots\dots\dots\dots\dots\dots (2)$$

Now the model we are considering is a model of overlapping generations, therefore, if now we have one worker and output is growing at the rate g, then, there must have been $1/1+g$ workers in the last period. Hence $a_o X_t = 1/1+g$. Substituting the value of X_t in (2) we get,

$$C_t = \frac{1-a_1(1+g)}{a_o(1+g)} \quad \ldots\ldots (3)$$

Note that C_t is the total consumption of $1/1+g$ working people and $1/(1+g)^2$ of retired people which takes place after the harvest X_t.

The price equation of the model is

$$1 = Wa_o + (1+r)\, a_1 \quad \ldots\ldots (4)$$

Where r is the rate of profit and W is the wage rate. Therefore, combining (3) and (4) we have four unknowns, namely, C_t, W, g and r but we have two equations.

Assume $W=W^*$, (which is a typical neo-Marxist closure)

As soon as W is known to us r will be derived from equation 4. Each individual will now allocate its corn wage W^* (because price is one in a one-commodity model) between consumption in the two periods of its economic life. Therefore, a consumer will maximize $U(C^1,C^2)$ subject to the budget constraint,

$$C^1 + \frac{C^2}{1+r} = W^*$$

Hence, we get C^1 (w^*,r) and C^2 (w^*,r). As individuals are identical in their preferences, we can derive total consumption of $1/1+g$ working people and $1/(1+g)^2$ of retired people, which is a function of g for given r and w^*. This functional relationship between C_t and g together with the functional relationship between C_t and g as defined in (3) will determine simultaneously C_t^* and g^* under a necessary and sufficient condition which ensures the existence of the equilibrium for a positive g (existence, uniqueness and stability of the equilibrium have been discussed in the appendix).

It may be observed that we have a negative relationship between r and g, as a reduction in r implies a change in the distribution of income in favour of the working people. Due to the positive marginal propensities to save of the working people, any change in the distribution of income in favour of the working people results in a rise in the savings, therefore, the rate of growth will rise.

In the neo-Marxist approach the relationship between g and r is positive, [see Marglin (1984)] because it has been assumed that entire wage income is consumed and entire profit is saved. In the neo-Classical model, growth was observed to be prior to distribution. Once g is fixed by the population growth rate (n), it is easy to determine the gross investment requirement for a given a_1. Once investment is known to us, equilibrating r can be determined from the saving schedule. However, the preference pattern helps us to determine present and future consumptions, which take place simultaneously with the distribution of income.

But in our model, growth and total consumption are simultaneously determined although, like neo-Marxist, distribution is prior to growth.

Another crucial characteristic of our model is that the equality between g and n is just an accident. However, if g is less than n, then unlike Harrod, it does not lead to a situation of growing divergence between g and n.

It may also be observed that the role of the closure $W=W^*$, to ensure the existence of unemployment, is not the same in our model and also in neo-Marxian model. Neo-Marxist´s argument on the cause of secular stagnation (i.e, $g < n$) was based on the idea of inadequate investment demand (because the closure $W = W^*$ determines the upper limit of r). Therefore, the neo-Marxian model will lead to the policies that will reduce the subsistence wage. Although in our model the saving-investment nexus is at the root of all troubles, it is not due to a `r´ which is less than the desired rate of profit for achieving full employment, but for the other way round. As in our model wage is the only source of savings, a fixed wage, which may be less than its desired value, causes unemployment. Hence our model leads to the policies that will stimulate savings via a higher wage rate.

Finally, any fiscal policy which lowers the rate of profit (r) affects the rate of growth in our model through substitution between present and future consumptions, where the price of one unit of future consumption is 1/1+r (it may be recalled that the wage rate is fixed by the closure). However, under Cobb-Douglas utility function , it will have no impact on the growth rate, because C-D utility function does not permit substitution between present and future consumption. If the tax revenue is distributed equally among the retired workers then present consumption of each retired worker will rise . This can be proved from the utility maximization exercise,(See appendix). Consequently growth rate will fall in our model with Cobb-Douglas utility function, as future consumption is the only source of savings.

Conclusion

Many non-neo-classical economists have attempted to bring both Marxian and Keynesian flavours into a single model of growth and distribution. Perhaps Robinson´s inflation barrier (1956, 1962) is one of them. However, no one has ever tried to synthesize Marxian and neo-classical ideas into a single model of growth with overlapping generations. Therefore, our model is a new inclusion in the literature of hybrid models of growth.

APPENDIX

A Suggestive Proof for Uniqueness, Existence and Stability of the Equilibrium

We shall prove existence, uniqueness and stability of the equilibrium with the help of a simple utility function, namely Cobb-Douglas utility function. One problem with Cobb-Douglas utility function is that it does not allow for substitution between present and future consumption, therefore, C.E.S. utility function can be used to study this, without any difficulty.*

Our objective is to max: $U(c^1, c^2)$ subject to the budget constrain

$$c^1 + c^2/1+r \quad = W$$

Considering a Cobb-Douglas utility function, the Lagrangian function becomes,

$$L = (c^1)^{\alpha} (c^2)^{\beta} - [c^1 + c^2/1+r \quad - W]$$

From this we get after substituting for W from (4)

$$C^1 = \alpha[1 - a_1/a_0 - r\ a_1/a_0]$$

* This exercise has been done in an extended version of this paper

Therefore, total consumption of 1/1+g current workers is

$$\alpha/_{1+g}\ [1-a_1/a_0 - r\ a_1/a_0\ ]$$

and total consumption of $1/(1+g)^2$ retired workers is $a_1(1+r)/a_0(1+g)$ [because they invested $a_1/a_0(1+g)$ amount of corn to produce x_t].

Therefore, $c_t = [\alpha/1+g(1-a_1/a_o) + a_1/a_o\ (1+g)]$
$+ r\ [a_1 - \alpha a_1/a_o(1+g)]$.............(5)

For a given $r=r^*$, we have a negative non-linear relationship between c_t and g, say $c_t=f(g)$. Let us denote the functional relationship of (3) by $c_t=\phi(g)$. It may be noted that f and ϕ curves have the following properties:

1. $f'<0,\ f''>0,\ \phi'<0,\ \phi''>0$

2. As $g \rightarrow \infty \Longrightarrow f(g) \rightarrow 0$

3. As $g \rightarrow 0 \Longrightarrow f(g) \rightarrow \alpha - \alpha a_1+a_1+r(a_1-\alpha a_1)/a_0$

4. As $g \rightarrow 0 \Longrightarrow \phi(g) \rightarrow 1-a_1/a_0$

5. As $g \rightarrow 1 -a_1/a_1 \Longrightarrow \phi(g) \rightarrow 0$

To ensure existence of the equilibrium for a positive 'g', we assume further that the intercept of the f curve with the c_t axis in the figure is smaller than the intercept of the ϕ curve with the c_t axis, in other words,

$$(1-a_1) > \alpha-\alpha a_1+a_1+r\ (a_1 - \alpha a_1)$$

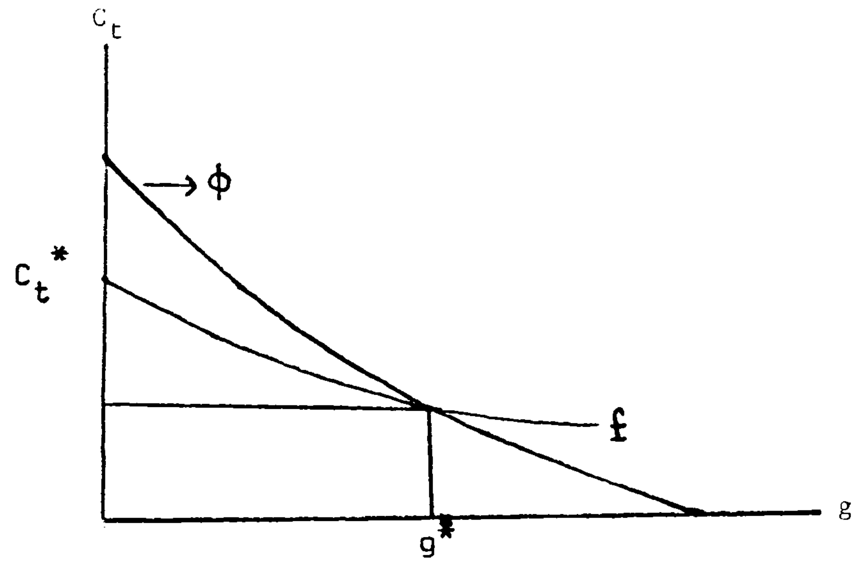

Fig. 1: Existence Of Equilibrium

Uniqueness

To arrive at a steady state solution f(g) must be equal to the function $\phi(g)$. Now if we equate these two functions, we will get a linear function in g. Therefore, we can have only one value of g. Furthermore, to ensure a positive g we need the intercept condition.

Local stability

It has been assumed in our paper that the consumption plans are determined before investment plans are laid. Since our model is an overlapping generations model, therefore, investment and hence the future growth rate g depends entirely upon the consumption plans of the current workers. Current workers in period t, i.e. 1/1+g, will get wage W^* [which is fixed by the closure] from the retired workers. Immediately they will decide their current and future consumptions through a utility maximization exercise and accordingly they will consume the amount in the current period and save rest of the amount for future consumption - which is the source of future growth rate. Note that retired worker's consumption is fixed because r is fixed and there is no bequest in our model. Therefore, the 'f' curve which is determined by the utility maximization exercise traces the actual total consumption of the current and retired workers. Hence there can not be any consumption point below or above the f curve for a given g, in or out of equilibrium. On the other hand ϕ curve shows the availability of total seed corn for current consumption if the economy grows at the rate g. Therefore, to the right of g^* (say g), actual consumption in period t is greater than what it should be if the economy wants to grow at the rate g in the next period also. Hence total investible resources is not sufficient to maintain the growth rate g. Therefore, the growth rate will fall and ultimately it converges to the growth rate g^*. Similarly to the left of g^*, total consumption is less than what is available. In this way, investment and hence the growth rate will rise. Hence the equilibrium is stable in our model. Now let us derive the time path of the variable g by the process of iteration. It may be seen that in our model $x_{t+1} = x_t(1+g_{t+1})$ $x_t = x_{t-1}(1+g_t)$etc., where $g_{t+1} \neq g_t$ for non steady state behaviour. Therefore g in the denominator of equation (5) is g_t because (5) shows the current plus retired workers consumption in period t. Since there is no bequest in our model, therefore, retired workers consumption is a function of the total investment made in period t-1 to produce x_t, therefore, it is a positive function of g_t. Similarly, total current workers employed to produce x_t is also a function of g_t.

We know that in a one commodity model x_t is always equal to the investment made in period t to produce x_{t+1} plus total actual consumption in period t. Therefore,

$$x_t = a_1 x_t (1+g_{t+1}) + c_t$$

Now from equation (5) we get actual consumption c_t, which is

$$c_t = A/a_o(1+g_t) \text{ where}$$

$$A = \alpha - \alpha a_1 + a_1 + r\ (a_1 - \alpha a_1)$$

Hence,

$$x_{t-1}(1+g_t) = a_1 x_{t-1}(1+g_t)\ (1+g_{t+1}) + A/a_o(1+g_t)$$

or,

$$1 = a_1(1+g_{t+1}) + A/a_o x_{t-1}(1+g_t)^2$$

Let us denote $1+g_{t+1}$ by v_{t+1} and $1+g_t$ by v_t. Finally we will get,

$$d\ V_{t+1}/dv_t = 2A/a_1\ a_o\ x_t\ V^2{}_t$$

which is always positive. If dv_t+1/dv_t is less than one [note that uniqueness and existence required $A<1$] then the system is stable in the sense that the phase path converges to the equilibrium value without oscillation. It may be observed that this is true only in the neighborhood of equilibrium. Furthermore, if dv_{t+1}/dv_t is less than one then it does not violate the intercept condition, required to ensure existence and uniqueness of equilibrium for a positive g.

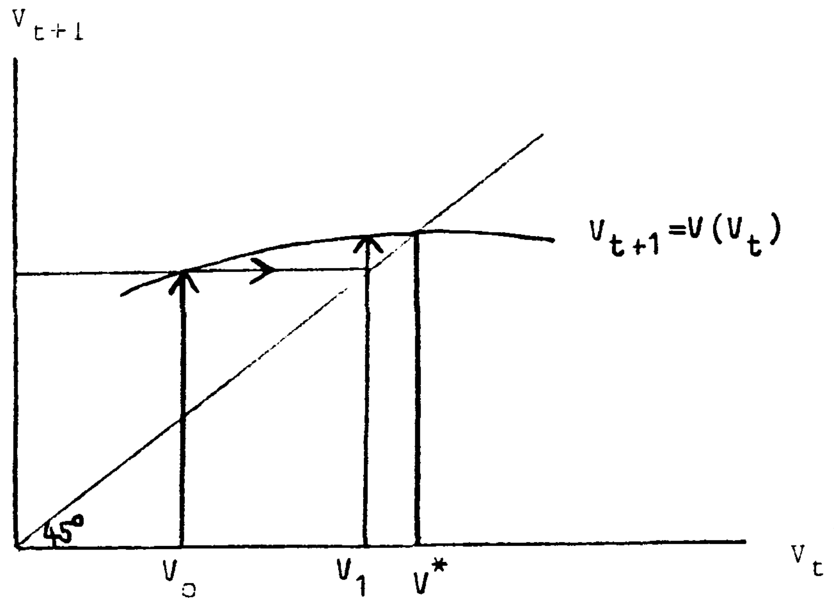

Fig. 2: Stability Of Equilibrium.

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