ON AGGREGATE MEASURES OF POVERTY

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1. Introduction

Measurement of degree of poverty in a society calls for (i) identification of the poor, and (ii) aggregation of their poverty characteristics. While the issue of identification is related to defining a poverty line, either an absolute or a relative one, the issue of aggregation relates to summarising the poverty characteristics into a single overall index.

This paper is concerned with the latter issue, viz., the construction of an index of poverty, given a poverty line. Starting with the seminal contribution of Sen (1976), while subjecting the traditional poverty measures as the head-count ratio and the poverty-gap ratio to extensive criticism, a number of new poverty indices have been proposed in the literature, e.g., Anand (1977), Blackorby and Donaldson (1980), Thon (1979), Kakwani (1980 a,b), Takayama (1978), Clark, Hemming and Ulph (1979), Hamada and Takayama (1978), Osmani (1978), Pyatt (1980), Fields (1980), Chakravarty (1980) and Foster et.al. (1984).

This paper is basically divided into two parts. In the first part we will briefly review some of the aggregate poverty measures that are closely aligned to the Gini coefficient of income inequality. These have been

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derived in an ordinal axiomatic framework that was first utilised in the derivation of a poverty index by Sen (1976). Earlier, Sen (1974) had also suggested an axiomatic framework for the Gini coefficient of income inequality.

In the second part, we take up the concept of censored income distributions and Takayama's index for a closer examination. Takayama (1979) has reconsidered, Sen's axiomatic framework and pointed out that the latter's axiomisation of the Gini coefficient needed an additional normalisation axiom. He provides this axiom, and then proceeds to define a new index of poverty based on the concept of a 'censored' income distribution. This new measure, it is claimed, is a closer translation of the Gini coefficient of income inequality into a measure of poverty.

In this paper, we attempt to establish that the Takayama index too is not fully axiomised; an additional normalisation axiom is required. Further, we propose that a 'slight' modification in the Takayama index gives rise to a new poverty index which can be considered to be an even 'closer' translation of the Gini coefficient of inequality into a measure of poverty. This modification provides a number of additional desirable properties, and it does away for all 'practical' purposes with a major criticism of the Takayama index <u>viz.</u>, its violation of the monotonicity axiom.

2. Absolute and Relative Aspects of Poverty

Sen (1981, p.22) observes: 'Poverty is, of course, a matter of deprivation. The recent shift in focus especially in the sociological literature - from absolute to relative deprivation is essentially incomplete as an approach to poverty, and supplements (but cannot supplant) the earlier approach of absolute dispossession'.

Attempts to capture absolute poverty are broadly related to (i) identification of the poor by defining an 'absolute' poverty line, and (ii) aggregation of their poverty characteristics without giving relative weights to their extent of poverty.

In defining an absolute poverty line one may adopt a biological approach that relates to minimum requirements for survival or work efficiency. Even here, the poverty line may get a 'relative' content due to significant variations in physical features, climatic conditions and work habits when making comparisons over communities, regions or countries.

Apart from nutritional or biological requirements, one may add some minimum social and cultural requirements in defining the absolute minimum of needs. This, of course, implies a greater 'relative' variation in the definition of the absolute minimum requirements when making inter-community or inter-country comparisons.

Relativity in the measurement of poverty is captured, to some extent, in the identification exercise itself by defining a 'relative' poverty line, as, for instance, designating a specified lowest per cent (say, 40 per cent) of the population in the national income distribution as poor, or declaring a given proportion (say, half) of the mean national income* as poor.

More generally, relativity is captured in the aggregation exercise. In the extreme, people take the view that poverty is an issue essentially of inequality only. Most of the measurement exercises, however, do not take this extreme view but rather attempt to 'reflect' to some extent the implications of inequality in their aggregation exercise. This is done generally by adopting some scheme of weighing the extent of relative deprivation of the poor.

Important differences in the formulation of such weighting schemes arise from the issue whether one should compare the poor only with the other poor or also with the non-poor.

If one ignores this 'externality' view of poverty that looks at the poor only from the viewpoint of the non-poor [see, Sen's (1981, p.9) succinct comments on a quotation from Rein (1971)], then there are essentially two views: a 'focus' on the poor alone and on the poor in society as a whole.

Different poverty measures accommodate these view-points in varying degrees; some use information on the non-poor to the extent of their numbers only; and, some, also use information on their incomes.

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* The first definition implies that a society will always have poor; but it expresses concern with the lowest income individuals of the society akin with Rawls' criterion of social welfare. In the second definition of the relative poverty line, it is not necessary that a society will always have some poor.

When poverty is viewed with respect to society as a whole, considerations of both an absolute as well as relative nature arise. If either the number or the mean income of the non-poor rises without any change in the number and income of the poor, should this society be considered less poor because its 'capacity' to ameliorate poverty has increased, or more poor, because the extent of the relative deprivation of the poor in comparison with the non-poor has increased? It appears that both should happen although the effects are in opposite directions; while the former refers to poverty of the society as a whole in an absolute sense, the latter refers to the poor relative to the entire community.

3. A Review of Poverty-Measures

In the ensuing discussion, we shall use the following notations:

n	=	total population
z	=	poverty line
m	=	number of poor, i.e., people below poverty line
y_i	=	income of the i^{th} individual
μ, μ_ρ, μ_π	=	respectively, mean incomes of the whole society; of the poor; of the non-poor.

For all the poverty measures under review here, it has been assumed that the poverty characteristic that is being aggregated is reflected in income. Incomes are arranged in a non-descending order, i.e.,

$$y_1 \leq y_2 \leq \cdots \leq y_m < z \leq y_{m+1} \leq, \cdots \leq y_n.$$

Nearly all the measures under review can be seen as normalised weighted sums of 'poverty gaps', viz.,

$$P = A \sum_{i=1}^{m} w_i \ (z-y_i) + B \qquad \ldots (1)$$

where $(z-y_i)$, is the 'poverty gap' of the i^{th} poor, w_i's are weights and A and B are normalisation constants.

3a. Head-count Ratio

The head-count ratio, H, is the proportion of persons below the poverty line in the whole population (=m/n). It is the traditional measure which is still being most frequently used by governments and international agencies, although it has been extensively criticised.

In terms of (1) above, H is obtained by putting $B=0$, $W_i=1$, $z-y_i=k$ (any constant) and defining $A \Sigma K = \frac{m}{n}$. In other words, it treats all poor and their incomes at par.

Its main deficiencies may be listed as follows:

i. It ignores distribution of income among the poor or their relative deprivation.

ii. It ignores the 'extent' of poverty, both individually and as an aggregate for the society as a whole.

iii. It ignores income characteristics of the non-poor of the society.

iv. It is transfer-insensitive, in the sense explained below.

If income is transferred from a poor to a rich person, poverty is not shown to increase. If income is transferred from a more poor to a less poor person, poverty is not shown to increase so long as the recipient remains below the poverty line. If he crosses the poverty line, the transfer leads to a reduction in poverty rather than an increase in it. If the transfer is from a poor to a non-poor, then there is no change in the index. Thus, as Sen (1981, p. 33) puts it, 'a transfer of income from a poor person to one who is rich can never increase the poverty measure H - surely a perverse feature'.

3b. Poverty-gap Ratio

The poverty-gap ratio is the aggregate of income shortfalls of poor persons from the poverty line divided by total income required for them to become non-poor (mz). Thus,

$$I = \sum_{i=1}^{m} (z - y_i)/mz = (z - \mu_p)/z = (1 - \mu_p/z) \ldots (2)$$

Viewed in terms of (1), it implies that B=0, w_i=1, and A=1. Some of its deficiencies are indicated below:

i. It ignores income distribution or relative deprivation among the poor.

ii. It ignores the number of proportion of people below the poverty line.

iii. It is insensitive to income transfers among the poor so long as nobody crosses the poverty line.

3c. Kakwani I

In this group of poverty measures that ignore relative deprivation of the poor, a variant has been suggested by Kakwani (1980a).

$$P \quad (\text{Kakwani I}) = m(z-\mu_p)/n\mu = \frac{1}{n\mu}\sum_{i=1}^{m}(z-y_i) \quad \ldots (3)$$

Thus, $P \quad (\text{Kakwani I}) = (H.I z)/\mu$

This measure is interpreted as the percentage of total income ($n\mu$) that must be transferred to the poor to bring them all above the poverty line. Interpreted as a weighted sum of poverty gaps, it implies all weights w_i(in eq. 1) to be equal, i.e., it ignores income distribution among the poor, as in H and I; it is thus, insensitive to transfers of incomes among the poor so long as nobody crosses the poverty line.

3.d. Sen Index

The poverty measure defined by Sen is given below.

$$P\ (\text{Sen}) = \frac{2}{(m+1)\ nz}\sum_{i=1}^{m}(m+1-i)\ (z-y_i) \quad \ldots (4)$$

Viewed as a normalised weighted sum of poverty gaps, it implies

$$A = 2/(m+1)nz;\ B=0;\ \text{and},\ W_i = m+1-i$$

The additional feature of this index is its weighting scheme, which ranks ordinally incomes of poor according to their relative deprivation among the poor. The axiomatic basis of this and some of the other measures will be discussed in the following section. Important features of this measure may be noted here.

i. It considers information on the non-poor as relevant in poverty measurement to the extent of their numbers only. It ignores the income characteristics of the non-poor, i.e., it is primarily based on the view that for the measurement of the degree of poverty in a society, one should only look at the incomes of the poor.

ii. The weighting scheme provides transfer sensitivity to the measure: if income is transferred from a poor to a higher-income poor, poverty would increase provided the richer person does not cross the poverty line.

Its transfer sensitivity may produce somewhat perverse results if the transfer of income from a poor to a richer poor enables the latter to cross the poverty line in the sense that the poverty measure may register a decrease in such a case.

The measure is related to the Gini coefficient of income inequality, and it can be written as

$$P\ (\text{Sen}) = H\left[1-(1-I)\left\{1-G_p m/(m+1)\right\}\right] \quad \ldots\ldots (5)$$

Where G_p is the Gini coefficient of income inequality among the poor, H, the head-count ratio, and I, the poverty-gap ratio.

For large number of poor, this reduces to

$$P\ (Sen) = H\left[I+(1-I)G_p\right] \qquad .....(6)$$

which can be written equivalently as

$$P\ (Sen) = H\left[1-\frac{\mu_p}{z}(1-G_p)\right] \qquad .....(7)$$

This measure can also be related to the Atkinson-Kolm concept of 'equally distributed equivalent (ede) income' of the poor when evaluated with respect to the Gini social evaluation function [See, Blackorby and Donaldson (1980)]. The ede income in this case may be written as $y_{ede} = \mu_p\ (1-G_p)$ and the poverty measure can be written as

$$P\ (Sen) = \frac{H}{z}\ (z-y_{ede}) \qquad ....(8)$$

This interpretation gives rise to a wider class of poverty measures arising from the 'ede' incomes of the poor relating to other social evaluation functions.

3.e. Anand Index

The Sen index has been modified by a multiplicative constant by Anand, giving rise to a new measure

$$P\ (Anand) = P\ (Sen)\ z/\mu \qquad ....(9)$$

The change refers to the normalisation procedure. This measure, by considering the mean income of the entire community, becomes sensitive to changes both in the incomes as well as the number of the non-poor; a rise in either of these will reduce the poverty measure. This is an advantage when one is looking at the degree of poverty in a community as a whole. But it also may be a demerit as a transfer of income from a poor to a rich person may not lead to an increase in the poverty measure; the positive effect of increased poverty gap on this person may be cancelled out by the changes in the number and incomes of the non-poor.

3. f. Kakwani Measures

Kakwani (1980a, b) has considered a number of alternative poverty measures. A family of measures proposed by Kakwani (1980a) can be written as

$$P \text{ (Kakwani II)} = \frac{m}{n\,\mu}\left[z - \mu_p\, f(G_p)\right] \quad \ldots(10)$$

Where, $0 \leq f(G_p) \leq 1$, $f'(G_p) < 0$, $f(G_p) = 1$, if $G_p = 0$
Kakwani gives a more specific form to this by considering $f(G_p) = (1-G_p)$, thus obtaining,

$$P\text{(Kakwani II 1)} = \frac{m}{n\mu}\left[z - \mu_p\,(1-G_p)\right]$$

$$= \frac{Hz}{\mu}\left[1 - \{\mu_p\,(1-G_p)\}/z\right]$$

$$= \frac{z}{\mu}\, P\text{ (Sen)} \quad \ldots(11)$$

Thus, this measure differs by the same multiplicative constant as the Anand measure, and is equivalent to the latter.

An alternative to this measure is also suggested in Kakwani (1980). By taking $f(G_p) = \frac{1}{1+G_p}$, the following measure is obtained.

$$P\ (\text{Kakwani II 2}) = \frac{H}{\mu} \quad \frac{z-\mu_p}{1+G_p} \qquad \ldots(12)$$

The weighting scheme is changed in a more general way by Kakwani (1980b) in a different contribution. The new (family of) measure (s) is given by:

$$P\ (\text{Kakwani III}) = \frac{m}{nz \sum_{i=1}^{m} (i)^k} \sum_{i=1}^{m} (m+1-i)^k\ (z-y_i) \ldots(13)$$

k can take any arbitrary value, k=1 being a special case. Alternative values of k permit the introduction of different levels of sensitivity of transfers at different levels of incomes among the poor, whereas k=1 treats transfers at all income positions among the poor as equally sensitive. K >1 would give a greater weight to income transfers at the lower end of incomes among the poor. When k=1, the normalisation constant of this measure, viz.,

$$\frac{m}{nz \sum_{i=1}^{m} (i)^k} \quad \text{reduces to} \quad \frac{2}{(m+1)nz}$$

and the measure thus translates into the Sen index.

Sen's index and its variants considered so far have all been concerned with the relative deprivation of the poor among the poor. The non-poor of the community appear via their numbers or incomes only in the normalisation constant.

We now consider two measures where the relative deprivation of the poor is considered not just among the poor but in relation to the entire community.

These measures are provided by Thon (1979) and Takayama (1979).

Thon's index may be written as

$$P\ (\text{Thon}) = \frac{2}{(n+1)nz} \sum_{i=1}^{m} (n+1-i)\ (z-y_i) \quad \text{..(14)}$$

Viewed as a normalised weighted sum, this measure differs from P (Sen) in defining the weights w_i as $(n+1-i)$ rather than $(m+1-i)$ indicating that if the number of non-poor in the economy increases, the heightened sense of relative deprivation would be reflected by an increase in the n in w_i. On the other hand, since n also enters into the denominator of the normalisation constant it would have an effect in the opposite direction.

3g. Takayama's Poverty Measure

Thon's measure can reflect changes in the number of non-poor persons, but it does not reflect the effect if their incomes go up as a whole either in the sense of increased relative deprivation for the poor or in the sense of increased capacity and therefore less poverty for the society.

The relative aspects of poverty are more adequately captured in the measure suggested by Takayama. This measure has the additional merit of being a very close translation of the Gini coefficient of income inequality into a measure of poverty. The poverty measure is defined as the Gini coefficient of inequality for the censored income distribution y_i^*, Where

$$y_i^* = y_i \text{ for } i = 1, \text{------}, m$$

and $$y_i^* = z \text{ for } i = m+1, \text{------}, n$$

The mean income of this distribution is given by

$$\mu^* = \left[m\,\mu_p + (n-m)z \right] / n = H\,\mu_p + (1-H)\,z$$

The poverty measure is defined as

$$P \text{ (Takayama)} = \frac{2}{\mu^* n^2} \sum_{i=1}^{n} (n+1-i)\,(\mu^* - y_i^*) \ldots (15)$$

where μ^* is the mean income for the censored distribution. If we replace y_i^* by y_i and μ^* by μ, we get the Gini coefficient for the actual income distribution.

This measure could also be written as

$$P \text{ (Takayama)} = \frac{2}{\mu^* n^2} \sum_{i=1}^{m} (n+1-i)(z-y_i) + (1+\frac{1}{n})(1-\frac{z}{\mu^*}) \;\ldots (16)$$

As such it can be taken as a special case of (1) where, $A = \frac{2}{\mu^* n^2}$, $B = (1+\frac{1}{n})\,(1-\frac{z}{\mu^*})$, and $w_i = n+1-i$

This measure is attractive in being a close translation of the Gini coefficient and in being able to capture some relative aspects of poverty. In particular, if the number of persons above the poverty line increases this would be reflected in the poverty index. This measure , however, is still insensitive to increases in income of the non-poor as neither the weighting scheme nor the scaling factor μ^* would be affected. Furthermore, it robustly violates a commensensical requirement on the poverty measure, viz., that if the income of a poor person falls, the measure should uniquely show an increase in poverty. The reason is that μ^* lies below z, and if the income of a person above μ^* but below z falls, i.e., moves closer to the censored mean income, equality in the censored distribution increases, and the coefficient of inequality, in this case the poverty measure, would actually show a decline.

A summary of the poverty measures reviewed so far is given in Table 1. In Table 2 the use of information on the poor and the non-poor by these indices is highlighted.

4. The Axiomatic Basis of some of the Poverty Measures

There are three kinds of axioms that have been utilised in the formulation and derivation of various poverty measures. These may be listed as 'legitimacy' axioms, 'ranking' axioms and 'normalisation' axio s. Although these are being listed here separately, it is only their integrated use in different combinations that leads to one or the other poverty measure.

Let $\underline{x}$ and $\underline{y}$ be two n-vectors of income where S is a set in the community of n people. Let x_i and y_i be the income of person i in the two cases, respectively, and let the poverty measures be such that $\underline{x}$ and $\underline{y}$ yield $P(\underline{x})$ and $P(\underline{Y})$ respectively, given z and S. Let $m(\underline{x})$ and $m(\underline{y})$ be the poor in S, respectively for $\underline{x}$ and $\underline{y}$.

TABLE 1

Poverty Measures as Normalised Weighted Sums of Poverty Gaps : $P = A \sum_{i=1}^{m} w_i (z - y_i) + B$

Measures	A	B	W_i
That ignore relative deprivation of the poor			
Head-count ratio	$\frac{1}{n(z-\mu_p)}$	0	1
Income-gap ratio	$\frac{1}{mz}$	0	1
Kakwani I	$\frac{1}{n\mu}$	0	1
that consider relative deprivation of poor among the poor only			
P (Sen)	$\frac{2}{(m+1)nz}$	0	(m+1-i)
P (Anand)	$\frac{2}{(m+1)\ n}$	0	(m+1-i)
P (Kakwani II)	$\frac{m}{nz\ \Sigma^{(i)}k}$	0	(m+1-i)
that consider relative deprivation of poor in the whole community			
P (thon)	$\frac{2}{(n+1)\ nz}$	0	(n+1-i)
P (Takayama)	$\frac{2}{\mu^{*}\ n^{2}}$	$(1+\frac{1}{n})(1-z/\mu^{*})$	(n+1-i)

TABLE 2

Use of Information on the Non-Poor

	Via the normalisation constant		via the weights on poverty gaps
	No.	Mean Income	
Head-count ratio	n	x	x
Income-gap ratio	x	x	x
Kakwani I	n	μ	x
P (Sen)	n	x	x
P (Anand)	n	μ	x
P (Kakwani II2)	n	μ	x
P (Kakwani III)	n	x	x
P (Thon)	n	x	n
P (Takayama)	n	x	n

The three groups of axioms can then be defined as follows:

4a. . Legitimacy axioms

Axiom M (monotonicity)

If for some $j \in \{m(\underline{x}) \cap m(\underline{y})\}$: $x_j > y_j$ and for all $i \in S$ such that $i \neq j, x_i = y_i$, then $P(\underline{x}) < P(\underline{y})$

This axiom implies that, given other things, a reduction in the income of someone below the poverty line must increase the poverty measure.

Axioms T1 (Weak transfer axiom)

If for some $j \in \{\{m(\underline{x}) \cap m(\underline{y})\} \cup \{(s-m(\underline{x}) \cap s-m(\underline{y})\}\}$ and $k \in m(\underline{x}) \cap m(\underline{y})$: $\geq x_j > y_j \geq y_k > x_k$ and $x_j - y_j = y_k - x_k$, and for all $i \in s$ such that $i \neq j, k$: $x_i = y_i$, then $P(\underline{x}) > P(\underline{y})$

This axiom says that a pure transfer of income to a poor person below the povery line from a richer person, without making either cross the poverty line, must reduce the poverty measure.

Axiom F (Focus axiom)

If $x_i = y_i$ for all $i \in m(\underline{x}) \cup m(\underline{y})$, then $P(\underline{x}) = P(\underline{y})$

This axiom accommodates the view that poverty measures must relate to the poor only and not respond to any changes in the conditions of the non-poor, i.e., of the society as a whole.

4b. Ranking Axioms

Axiom R1 The weight w_i on the poverty gap of person i equals the rank order of i in the interpersonal welfare ordering of the poor, i.e., $w_i = m+1-i$

Axiom R2 The weight w_i on the poverty gap of person i equals the rank order of i in the interpersonal welfare ordering of the entire community, i.e., $w_i = n+1-i$

Axiom R3 The weight w_i is a function of the relative rank of the poor among the poor, $w_i = (m+1-i)^k$

4c. Normalisation Axioms

Axiom N1 If all the poor have the same income, the poverty measure is equal to HI.

Axiom N2 If all the poor have the same income, the poverty measure is equal to $\frac{H(Z-{}^{\mu}P)}{n\,\mu}$

Axiom N3 If there are no poor in the community, the poverty measure is equal to zero.

Axiom N4 If all the poor have no incomes, the poverty measure is equal to H.

Axiom N5 If all the poor have the same non-zero income equal to μ_p, then the poverty index is the Gini coefficient of income inequality between poor and non-poor as groups.

5. A Reconsideration of Takayama's Index

P (Takayama) can be viewed in two ways:

i. as a normalised weighted sum of poverty gaps; and

ii. as the Gini coefficient of income inequality of a 'censored' income distribution.

This censored income distribution is obtained by truncating the income distribution at the poverty line assigning a value z to all non-poor incomes. In the derivation of his index, Takayama (1979) uses Axioms M, R2, N3 and N4.

In the first part of his paper Takayama considers Sen's (1974) axiomisation of the Gini coefficient of income inequality with the help of these axioms and establishes that Sen's axiomisation is not complete unless an additional normalisation axiom is added.

An attempt will be made here to establish that Takayama's derivation of his poverty index is also not fully axiomised unless an additional normalisatiom axiom is added. His axioms give rise to not one but a family of poverty measures and his index is a special case of this family of indices. His own measure violates the monotonicity axiom whereas a range can be specified in this family of measures where this axiom would not be violated.

Consider now a class of censored income distributions, $y^*(z,k)$

where $y_i^* = y_i$ for $i=1, —— m$ (i.e. $y_i < z$)

$y_i^* = z+k$ for $i=m+1, —— n$ (i.e. $y_i \geq z$) ...(17)

Let μ_k^* be the mean income of the censored income distribution for any given value of k, and let k be any non-negative arbitrary number.

It can be shown that the poverty index

$$P = \frac{2}{\mu_k^* n^2} \sum_{i=1}^{n} (n+1-i)\,(\mu_k^* - y_i^*) \qquad ...(18)$$

for any value of k is consistent with the four axioms introduced above. In other words, an additional normalisation axiom will be needed to uniquely define the value of k.

One difficulty of the Takayama index noted by Sen and Takayama himself is the robust violation of the requirement that a reduction of income of someone close to z but still poor, should lead to an increase in the poverty measure. In Takayama's measure, the index may actually go down. The reason for this is that in the censored income distribution, the Gini coefficient of inequality decreases if someone's income which is above the censored mean income (μ_k^*) moves closer to it. In Takayama's measure, this arises because (μ^*) is less than z by definition.

It will be observed that this violation would not occur beyond a certain range of values for k. If k is so chosen that (μ_k^*) lies above the poverty line z, then any reduction in the income of a relatively 'rich poor' would still lead to a fall in the poverty index.

The value of k which we require for this purpose should satisfy the following condition:

$$\left[m\,\mu_p + (n - m)\,(z + k)\right] / n > z$$

or

$$k > H\,(z - \mu_p)/(1 - H)$$

6. <u>A Modified Measure of Poverty</u>

Consider now a censored distribution given by

$$y_i^* = \mu \text{ for } i = m+1, \text{ ---}, n, \text{ and}$$

$$y_i^* = y_i \text{ for } i = 1, \text{ ---}, m$$

it implies a value of $k = \mu - z$

The Gini coefficient of income inequality for this censored distribution is

$$P = \frac{2}{\mu n^2} \sum_{i=1}^{n} (n+1-i)\,(\mu - y_i^*) \qquad \ldots(19)$$

where μ is the mean income of the actual as well as the censored distribution.

We propose P as the modified measure of poverty.

This measure is derived by defining the poverty index as a normalised weighted sum of poverty gaps of the poor from the poverty line and can be written either as

$$P = A \sum_{i=1}^{m} w_i \, (z - y_i) + B \qquad \text{....(20)}$$

or as

$$P = S \sum_{i=1}^{n} w_i \, (\mu - y_i^*) + T \qquad \text{...(21)}$$

It would satisfy axioms M (monotonic welfare), R2, N3, and N4. These axioms are the same as the ones used in Takayama's index. In addition we need N5.

The derivation of the index is as follows. From axiom N3, if no persons are below the poverty line, then the poverty index is zero.

This axiom specifies the value of T=0 in (21). For, if all persons are non-poor,

$$y_i^* = \mu_\Pi \text{ for all } i, \text{ and } \mu = \mu_\Pi$$

Thus,

$$P = \sum_{i=1}^{n} w_i \, (\mu_\Pi - \mu_\Pi) = 0$$

in order to ensure that P=0, T must be equal to zero.

Axiom N4 states that if all poor have an income equal to zero, the poverty measure is equal to the head-count ratio H ($= {}^{m}/n$). This axiom is used to derive the value of S in (21), once axiom R2 is used to specify the weights $w_i = (n+1-i)$.

If all the poor have zero incomes, the mean income of the population is

$$\mu = \frac{(n - m)}{n} \mu_{\Pi} \qquad \text{or } \mu_{\Pi} = \frac{n}{n - m} \mu \quad \ldots (22)$$

In this case,

$$P = S \sum_{i=1}^{m} (n+1-i) + S \sum_{m=1}^{n} (n+1-i) \left(\mu - \frac{n}{(n-m)} \mu \right) \quad (23)$$

This solves to

$$P = \frac{S \mu m n}{2} \quad \ldots (24)$$

Since by axiom N2, P = m/n, in this case, we have

$$S = \frac{2}{\mu n^2} \quad \ldots\ldots (25)$$

These axioms thus provide the measure P, which may be written as

$$P = \frac{2}{\mu n^2} \sum_{i=1}^{n} (n+1-i) (\mu - y_i^*) \quad \ldots . (26)$$

This index can also be written as

$$P = \frac{2}{\mu n^2} \sum_{i=1}^{m} (n+1-i) (z-y_i) + \frac{2}{\mu n^2} \sum_{i=m+1}^{n} (n+1-i) (\mu - \mu_{\Pi})$$

or

$$P = \frac{2}{\mu n^2} \sum_{i=1}^{m} (n+1-i)(z-y_i) + \frac{2}{\mu n^2} \sum_{i=1}^{m} (n+1-i)(\mu - z) + \frac{2}{\mu n^2} \sum_{i=m+1}^{n} (n+1-i) (\mu - \mu_{\Pi}) \quad \ldots\ldots\ldots (27)$$

The latter two terms add up to give S in (21) while $A = 2/\mu n^2$. Thus, the measure P can be seen as a normalised weighted sum of the poverty gaps of the poor.

Since for any value of k in (20), the Gini coefficient of inequality in the censored distribution can be written as a normalised weighted sum of poverty gaps of the poor, we use axiom N5 to uniquely define our index.

In the case where all poor have non-zero incomes equal to μ_p, we have

$$P = \frac{2}{\mu n^2} \sum_{i=1}^{m} w_i (\mu - \mu_p) + \frac{2}{\mu n^2} \sum_{i=m+1}^{n} w_i (\mu - \mu_\Pi) \quad ..(28)$$

or

$$P = \frac{2}{\mu n^2} \left[(\mu - \mu_p) \sum_{i=1}^{m} w_i + (\mu - \mu_\Pi) \sum_{i=m+1}^{n} w_i \right] \quad ....(29)$$

The expression on the right is interpreted as the Gini coefficient of inequality between poor and non-poor as a group (say, G_b), and this expression will be obtained only when $z+k = \mu_\Pi$ in (17)

6a. Some Properties of the Modified Measure

In defining the modified measure of poverty, we use the concept of censored income distribution as in Takayama (1979). In his censored distribution, however, all incomes of the non-poor are treated at the poverty threshold z. In our case all incomes of the non-poor are located at μ_Π, the mean income of the non-poor. The essential point in both procedures is to ignore income differences among the rich. Whereas changes in the number of rich persons would be reflected in the Takayama index, in our measure, changes in both the number as well as the incomes of the rich are reflected.

For purposes of comparison, consider the following two expressions for the Takayama index and our index.

$$P\ (\text{Takayama}) = \frac{2}{\mu^* n^2} \sum_{i=1}^{m} (n+1-i)\ (\mu^* - y_i) + \sum_{i=m+1}^{n} (n+1-i)(\mu^* - z) \quad ..(30)$$

$$P\ (\text{modified}) = \frac{2}{\mu n^2} \sum_{i=1}^{m} (n+1-i)(\mu - y_i) + \sum_{i=m+1}^{n} (n+1-i)(\mu - \mu_{\Pi}) \quad ..(31)$$

The advantage in our measure is that it is able to capture more adequately both the relative and the absolute aspects of poverty. One can mention three aspects in this context that a good index of poverty should be able to capture. First, if the number of the non-poor increases, poor will feel poorer in the welfare ranking. This aspect is covered both in the Takayama index and our index and it is achieved by deriving the weights from the whole population ($w_i = m+1-i$) rather than just from the poor ($w_i = m+1-i$). Second, if the number of poor does not change but their incomes increase, then two considerations arise. First, there is an increase in the capacity of the economy to solve the poverty problem. In an absolute sense, the society as a whole should look less poor. This is captured by our given coefficient by the term in $2/\mu n^2$. As μ increases, poverty should decline. This is, however, not captured in the Takayama index where correspondingly the term μ^* occurs which is the mean income of censored distribution and would not be affected by changes in incomes above the poverty line. The second aspect is that as the incomes of the non-poor increase, the sense of relative deprivation of the poor increases. From this relative point of view the poverty index should go up. This aspect is captured in our index by the term $(\mu - y_i^*)$ whereas the corresponding term in the Takayama index $(\mu^* - y_i^*)$ is unable to capture it. The reason is that μ^* is defined as $\mu^* = H\mu_p + (1-H)z$ and it is invariant with respect to any changes in the non-poor incomes.

The proposed measure is a straightforward extension of the Gini coefficient into a poverty measure. By replacing μ^* with μ, the true mean income of the population, it is actually a closer translation of the Gini coefficient of income inequality into a measure of poverty than the Takayama index. Furthermore, as long as μ is above z, any reduction in the income of a poor person would lead to a reduction in the poverty measure.

6b. A Decomposition of the Poverty Index

For a decomposition of the measure, consider the following

$$P = A \sum_{i=1}^{n} w_i \, (\mu - y_i^*)$$

$$= A \sum_{i=1}^{m} w_i \, (\mu - y_i) + A \sum_{i=m+1}^{n} (\mu - \mu_{\pi})$$

$$= A \left(\sum_{i=1}^{m} w_i \, (\mu - \mu_p) + \sum_{i=m+1}^{n} w_i \, (\mu - \mu_{\pi}) \right) +$$

$$A \left(\sum_{i=1}^{m} w_i \, (\mu_p - y_i) \right) \qquad \ldots.(32)$$

The first term on the right is G_b, the Gini coefficient of income inequality between rich and poor as a group.

This enables us to write

$$P = G_b + W \quad G_p \qquad \dots\dots\dots\dots(33)$$

where G_p is the Gini coefficient of income inequality among the poor, and

$W = \frac{m \, (\mu_p \cdot m)}{n \, (\mu \cdot n)}$, is the income share of the poor multiplied by the population share. Suppose ϕ is the income share of the poor. Then

$$P = G_b + H. \ \emptyset. \ G_p \qquad \dots\dots\dots(34)$$

Notice that G_b can be written as H-Ø from the following:

$$G_b = \frac{2}{\mu n^2} \sum_{i=1}^{m} (n+1-i) \ (\mu - \mu_p) + \sum_{m+1}^{n} (n+1-i) \ \{\mu - \frac{(n\mu - m\mu_p)}{n-m}\}$$

This simlifies to

$$G_b = \frac{m}{n} \left[1 - \mu_p / \mu\right] \qquad \dots\dots\dots(35)$$

Thus $P = H - \emptyset + H\emptyset \ . \ G_p$ ....(36)

If I is the poverty gap ratio, we can write

$$1 - I = \mu_p / z \qquad \dots\dots\dots(37)$$

The poverty index could then be written from (32) and (33) as

$$P = H \ (1 - \frac{\mu_p}{\mu}) \ + H. \ I. \ G_p \qquad \dots\dots\dots(38)$$

This simplifies to

$$P = H\left[\gamma + I\,(1-\gamma) + \phi\, G_p\right] \qquad \text{........(39)}$$

where $\gamma = (1-z/\mu)$

We may compare this to the decompositions of their respective poverty measures suggested by Sen and Takayama. We have, for Sen's measure of poverty

$$P\ (\text{Sen}) = H\left[I + (1 - I)\ G_p\right] \qquad \text{.........(40)}$$

and for the Takayama index,

$$P\ (\text{Takayama}) = H\left[(1 - \phi)\ I + \phi\ G_p\right] \qquad \text{..........(41)}$$

In both these measures, poverty is seen to depend on factors H, I and G_p, respectively, the head-count ratio, the poverty gap ratio and the Gini coefficient of income distribution among the poor. In addition to using these factors, we have another factor $\gamma = (\mu-z)/\mu$, which is the gap between mean income and the poverty-line relative to the mean income and can be taken as reflecting the capacity of the economy to ameliorate poverty.

In (39), the first term inside the brackets reflects capacity, the second, the aggregate poverty gap and the third, the distribution of income among the poor.

7. Summary

In this papaer we have reviewed the indices of poverty based on the Gini index which have been proposed in recent literature on the subject following the seminal work of Sen (1976). We have then suggested a new index of poverty which is based on the concept of censored income distributions used by Takayama (1979) and Takayama and Hamada (1978). It is shown that this new measure is a closer translation of the Gini index of income inequality into a measure of poverty and it is able to capture the relative and absolute aspects of poverty more adequately than most other poverty indices related to the Gini coefficient of income inequality. A useful scheme of decomposition of this measure is also suggested and it is compared with the decompositions of the Sen and Takayama indices.

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