Asymmetric Agents and Dynamic Contributions to Public Goods

Presentation at NIPFP

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February 15, 2018

Introduction

A provision of a public good often benefits heterogeneous individuals asymmetrically. Examples:

- Discrete public good: an elevator on a multi-storied building that is conjointly used by individuals residing on different floors.
- Continuous public good: quality of air we breathe in. A given improvement in air quality benefits individuals asymmetrically depending on their respective health conditions, their ages and their vulnerability due to other factors.

Definition: A continuous public good can be accessed at any level of provision whereas a discrete public good can *only* be accessed after the project is complete and provision is ready for utilization.

Central idea

- Individuals contribute towards the provision of *continuous* and *discrete* public goods over time. Contribution costs are symmetric across individuals.
- Individual contributions advance the project: either progress the provision (continuous) or, brings the completion time forward (discrete), and thus increase the lifetime benefits for everyone.
- However, an increase in collective contributions benefits individuals asymmetrically, and thus provides differentiated incentives to contribute.
- Individual contributions are based on cumulative collective contributions in equilibrium, and we analyse the role valuation asymmetry plays in dynamic contributions of asymmetric individuals.

Literature: homogeneous agents

- Admati and Perry (1991) analyse joint contributions to a public project without commitment, and showed individuals make small *step-by-step* contributions and increase their contributions as the project progresses. However, some socially beneficial projects *do not* get completed: delay in completion due to small initial payments.
- Kessing (2007) confirms the findings of *strategic complementarity* and social inefficiency in continuous time.
- Fershtman and Nitzan (1991) showed that homogeneous individuals' contributions towards a *continuous* public good decrease as the project progresses, and thus, the contributions are *strategic substitutes*. The *free-rider problem* aggravates as the agents make their contribution decisions based on *cumulative collective contributions*.

Literature: asymmetric agents

- Compte and Jehiel (2003) consider dynamic contributions towards a *small* discrete public project by agents who values the provision asymmetrically. They show, the project completes in two large contributions, where the individual with a higher valuation makes the last and the largest one-time contribution. Moreover, all socially beneficial projects get completed in equilibrium.
- Bhattacharya et. al (2017) consider heterogeneous agents who differ in their level of *impatience*. They showed that in an *Markov perfect equilibrium*, when the individuals differ in their levels of impatience beyond a *threshold*, contribution of a relatively impatient people become *strategic substitutes across time* whereas contribution of a patient individual remain *strategic complements*.

Contributions of this paper

- Dynamic contributions of asymmetric individuals are *strategic complements* in case of both *continuous* and *discrete* public goods.
  Remark: In sharp contrast with Fershtman and Nitzan (1991), and extends the result of Compte and Jehiel (2003) in continuous time for a project of any size.
- In case of a *discrete* public good, an individual with a higher valuation contributes a greater amount every period in the asymmetric completion Markov perfect equilibrium. The equilibrium still exhibits social inefficiency. I completely characterize the socially beneficial projects that are *not* completed in equilibrium.

Continuous good and partial free-riding

- As the valuation asymmetry exceeds a threshold, the lower valuation individual *does not* contribute until a critical provision is guaranteed from the contributions of the higher valuation individual, and thus partially free-rides on the higher valuation individual.
- When the valuation asymmetry lies within a threshold, incidence of such a partial free-riding behavior is restricted, but the project needs a critical external funding to get started. In a benchmark case of homogeneous agents, the project always needs an external support and no partial free-riding is observed.

Remark: As the valuation asymmetry widens, the need for a critical external support diminishes with an increasing incidence of partial free-riding behavior in terms of non-contribution. However, having an individual with a high valuation benefits the society.

Policy implication: a charitable giving example

In the spring of 1995, Wisconsin Governor Tommy Thompson offered USD 27 million in state bonds to finance a new USD 72 million basketball arena for the University of Wisconsin, on the condition that the rest of the money be raised by private donations. A few days later, on April 1, 1995, Wisconsin's U.S. Senator Herb Kohl, who is also a wealthy entrepreneur, pledged USD 25 million to the project, which would now be called the Kohl Center. On June 27, 1995, Ab Nichols, a former University of Wisconsin basketball star, pledged USD 10 million. In November of 1995 the Kellner family pledged USD 2.5 million. By the time the university formally announced its public fund-raising campaign in February of 1996, it needed only USD 7 million to reach its goal. (Andreoni, 1998, Journal of Political Economy)

Model: Contribution cost

- At every instant t, an individual i chooses to contribute $x_i(t), i \in \{1, 2\}$ towards the provision of public good that maximises the present value of her lifetime pay-off.
- Contribution costs are symmetric and is represented by an increasing and convex function: $C_i(x_i(t)) = \frac{c}{2} x_i(t)^2$. Marginal cost of an additional contribution is increasing and symmetric across individuals.
- Individual contributions progresses the state of the project.

"state" of the project

- The state of the project at an instant t is defined by the *cumulative collective contributions*, $k(t) = \sum_t \sum_i x_i(t), i \in \{1, 2\}$.
- The state of the project progresses by individual contributions, but also get depreciated at a rate $\delta > 0$, captured by the *law of motion*: $\dot{k} = \sum_i x_i(t) - \delta k$.
- If the good is continuous, provision can be accessed at any stage and therefore accumulated contributions $k(t)$ represent the provision at any t.
- If the good is discrete, contributions are immediately sunk and the provision can be accessed only after completion time $T = inf\{t \geq 0 : k(t) = K\}$ when collective contributions add up to project cost, K.

Central idea: valuation asymmetry

Individuals derive asymmetric benefits from the provision.

- Continuous public good: the benefits derived by the agents are functions of *cumulative contributions* (representing the provision at any t) $k(t)$. The benefit function of an agent i is linear, exhibits constant marginal benefit, given by $f_i(k) = a_i k$, where $a_1 \neq a_2$.
  Note: an increase in contribution by an agent benefits everyone asymmetrically and thus provides them with differentiated benefits to contribute.
- Discrete public good: an exogenous stream of benefits D_i flows for each agent i from the completion time T onwards, and $D_1 \neq D_2$.
  Note: An increase in contribution by an agent brings the completion time $T = inf\{t \geq 0 : k(t) = K\}$ forward and thus increases discounted lifetime benefits asymmetrically for all.

Dynamic optimization problems

- Continuous public good: An individual chooses $x_i(t)$ to

$$\max_{x_i} \int_0^\infty e^{-rt}(a_i k - \frac{c}{2}x_i^2)dt \tag{1}$$

subject to $\dot{k} = \sum_i x_i - \delta k$, and $k(0) = 0$.

- Discrete Public good: An individual chooses $x_i(t)$ to

$$\max_{x_i} \; -\int_0^T \frac{c}{2}x_i^2 e^{-rt}dt + \int_T^\infty D_i e^{-rt}dt \tag{2}$$

subject to $\dot{k} = \sum_i x_i - \delta k$, $k(0) = 0$ and $k(t) = K$, where $T \equiv inf\{t \geq 0 : k(t) = K\}$.

Markov perfect equilibrium (MPE)

I focus on *Markov perfect equilibrium* wherein individual contributions $\phi_i(k)$ and the value functions $V_i(k)$, representing maximum lifetime benefits, depends on the *cumulative collective contributions*, k. An MPE is *subgame-perfect* by definition.

Definition

An *MPE* is defined by a strategy profile of all the agents $(\phi_1^*(k), ..., \phi_m^*(k), ..., \phi_n^*(k))$ such that maximum discounted lifetime pay-off $V_m(\phi_1^*(k), \cdots, \phi_m^*(k), \cdots, \phi_n^*(k)) \geq V_m(\phi_1^*(k), \cdots, \phi_m(k), \cdots, \phi_n^*(k))$, for all $m \in \{1, 2, ..., n\}$.

Result: Continuous Public Good

Proposition

(i) *The contribution strategies of two individuals that constitute a Markov perfect equilibrium are given by:* $\phi_1(k) = \frac{1}{c}(\beta_1 + \gamma k)$, *and* $\phi_2(k) = \frac{1}{c}(\beta_2 + \gamma k)$, *where* $\beta_1 = \frac{3a_1}{r-\delta} - \frac{2(\delta+r/2)}{r-\delta}\frac{\delta(2a_1-a_2)+r(a_1+a_2)}{\delta(\delta+2r)}$, $\beta_2 = -\frac{\delta(2a_1-a_2)+r(a_1+a_2)}{\delta(\delta+2r)}$ *and* $\gamma = \frac{2c}{3}(\delta + r/2)$.

(ii) *Dynamic contributions of asymmetric individuals towards the provision of a continuous public good are strategic complements across time.*

(iii) *The steady state value of the provision of the public good is given by:* $k^{SS} = \frac{3(a_1+a_2)}{\delta c(\delta+2r)}$.

Partial free-riding

Proposition

Consider the case $r < \delta$.

(iv) *For $\frac{a_1}{a_2} > \frac{2\delta+r}{\delta-r}$, where $\frac{2\delta+r}{\delta-r} > 1$, we have $\beta_1 > 0$ and $\beta_2 < 0$, which implies that $\phi_1(k) > 0$, $\forall k$, whereas $\phi_2(k) > 0$, $\forall k > \tilde{k} = \frac{3a_1(2\delta+r)+3a_2(r-\delta)}{2c\delta(\delta+2r)(\delta+r/2)}$.*

(v) *For $0 < \frac{a_1}{a_2} < \frac{\delta-r}{2\delta+r}$, where $\frac{\delta-r}{2\delta+r} \in (0,1)$, we find $\beta_1 < 0$ and $\beta_2 > 0$, which implies $\phi_1(k) > 0$, only when $k > \hat{k} = \frac{3}{2c(\delta+r/2)}[\frac{2\delta+r}{r-\delta}\frac{a_1(2\delta+r)+a_2(r-\delta)}{\delta(\delta+2r)} - \frac{3a_1}{r-\delta}]$, whereas $\phi_2(k) > 0$, $\forall k > 0$.*

(vi) *For $\frac{\delta-r}{2\delta+r} < \frac{a_1}{a_2} < \frac{2\delta+r}{\delta-r}$, we have both $\beta_1 < 0$ and $\beta_2 < 0$, which implies that $\phi_1(k) > 0$, $\forall k > \hat{k}$, and $\phi_2(k) > 0$, $\forall k > \tilde{k}$, and as a result, the project needs a critical external contribution of $\min\{\hat{k}, \tilde{k}\}$ to get started.*

Result: Discrete Public good

Proposition

(i) *The contribution strategies of the two individuals that constitute an asymmetric completion MPE are given by:* $\phi_1(k) = \frac{1}{c}(\beta_1 + \gamma_1 k)$ *and* $\phi_2(k) = \frac{1}{c}(\beta_2 + \gamma_2 k)$*, where* $\beta_i = \frac{-rcK+\sqrt{6cD_i}}{3}$*, and* $\gamma_1 = \gamma_2 = \frac{rc}{3}$*,* $i \in \{1, 2\}$*.*

(ii) $\gamma > 0$*, i. e., individual contributions are strategic complements across time.*

(iii) *Individual i always renders a positive contribution as long as* $D_i > \frac{cr^2K^2}{6}$*. Moreover,* $\phi_1(k) > \phi_2(k)$*,* $\forall k > 0$ *as long as we have* $D_1 > D_2$*, i.e., the individual with higher valuation contributes a higher amount every period.*

(iv) *Some socially beneficial projects, characterized by* $\frac{D_1+D_2}{K^2} \in \{\frac{cr^2}{4}, \frac{cr^2}{3}\}$*, are not completed in the MPE.*

Discussion

- Individuals increase their contributions as the project progresses, because their incentive to contribute is more when the project is closer to completion.
- In case of discrete public good, an increase in collective contribution brings the completion time forward increasing lifetime benefit asymmetrically for the agents. An agent finds it optimal to increase her own contribution in response and further expedite the completion.
- Increase in lifetime benefits due to an earlier completion is greater for a higher valuation individual which prompts her to contribute a larger amount every period to further expedite completion.
- Small initial payments cause delay in completion which leads to social inefficiency.

Discussion

- *Strategic complementarity* in case of continuous public good is in sharp contrast to Fershtman and Nitzan (1991).
- Since marginal benefit is positive, an increase in collective contribution (or the provision) benefits everyone. A higher valuation individual contributes from the beginning primarily because utilization of that provision returns a significant accumulated benefits to her, making the contribution incentive compatible. An individual with a significantly lower valuation finds it incentive compatible to contribute only after the contributions accumulate to a threshold, so that the lifetime benefits become worthy of the contribution cost.
- When none of the individuals derive significant benefits, that critical guarantee in provision is needed in terms of external contribution.

Contributions and policy implications

- Dynamic free-rider problem is alleviated with a constant marginal benefit function.
- *Partial free-riding* offers a novel point of view towards the free-rider problem in case of continuous public good. The external funding guarantee, if effectively utilized, can provide incentives to contribute, and increase contributions over time, towards a public good. Even this partial free-riding turns out to be socially beneficial.
- Asymmetry in valuation provides individuals with differentiated incentives and yields (mostly) positive social outcomes.
- The result in discrete public good extends the Compte and Jehiel (2003) work in continuous time without any restriction in project size.