Endogenous Leadership in a Federal Transfer Game

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Abstract

Conventional wisdom suggests that, to negate fiscal externalities imposed by provinces which spend too much and raise lower local resources, central authority should always be a first mover in the transfer game. In spite of such recommendations, central governments, in almost all countries, chooses to be the second mover from time to time. We explore the conditions, other than the familiar political economy arguments, under which the central government optimally chooses to be the second mover. Moreover, ex post transfer protocols may induce provinces to generate more local resources than otherwise. The results depend crucially upon the benefit received by each level of government from the project outcomes of other tier.**

Key words: Federalism, Transfer Game, First and Second Mover Advantages

JEL classification codes: H77, H23, H10.

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1. Introduction

In this paper, we attempt to answer the following set of questions: in order to maximize its benefit, should a central government in a federal economy choose to make *ex post* grants (disbursed after the provinces make their tax-expenditure decision) to provinces? Or should it commit to an *ex ante* transfer and try to control provincial action? Why does the former behaviour persist in real world in spite of the fact that recent intellectual paradigm favors the latter?

Due to inherent vertical and horizontal imbalances in a federal economy, central (or federal) authority has to transfer funds to provinces. A point of concern is the degree of control wielded by the central government over the subnational units regarding disbursement and utilization of such funds. The debate goes back to the foundation of USA, the first federal (as per the current usage) country in modern world. Madison (1887) argued that, in a heterogeneous country, some freedom for the local level of government is necessary so that they can choose their level and composition of public good in an effective manner. Others (Hamilton, *ibid*.1887) have raised the fear that such freedom will induce higher provincial budget deficits (encouraging lower taxes) and subsequent bailouts by Centre can prove disastrous for the federation as a whole.

One could say that Hamilton's fear was realized in Brazil and Argentina during the 1990's. In Europe, significant bailouts occurred in post-war Germany in the provinces of Bremmen, Saarland and Berlin (Rodden, 2006). The issue has assumed importance in the light of recent events in Eurozone (a quasi federal setup). The Hamiltonian paradigm places emphasis on fiscal solvency at various levels of Government, including the Centre. Thus, the only benefit that accrues to Centre is the savings (to be spent in various central projects), net of transfer to the provinces (which is used to finance provincial budget) out of a fixed revenue resource base. Thus fiscal stress in provinces (and, unchecked actions of the provinces are likely to lead to such eventuality), will ultimately be transferred to the Centre and the latter (or the nation as a whole) will suffer. For theoretical analyses of such "common pool" problem, see Wildasin (1997), Velasco (2000) and Goodspeed (2002).

As already noted, recent policy prescriptions are heavily biased to Hamiltonian paradigm. Anwer Shah (2006, p. 47), for example, prescribes that "Grants to finance subnational deficits, which create incentives for running higher deficit in future" are to be avoided. In India, the Twelfth Finance Commission, a constitutional body which regulates Centre-state financial relation to a large extent, had recommended (2004) the termination of central governments' role in assuming the states' debt. In other words, Centre should dispense with the *ex post* grants.

In spite of this intellectual onslaught, bailout by Centre (which is, almost and always, *ex post* in nature) still persists, ostensibly, in the name of equity. One can argue that such transfers are inherently politically motivated. Evidence of political motivation behind *ex post* grants, either partisan or strategic, is well documented in, say, Solé-Ollé and Sorribas-Navarro (2008) or Arulampalam et al. 2008).

What we attempt in this paper is to explain the persistence of such *allegedly* inefficient behavior from efficiency calculations. Due to sequential nature of grant dispensation in a federation, the cost-benefit calculus (of ex post vis-à-vis ex ante) should be done in a sequential game framework, in which the provinces and the Centre are the two sets of players. Depending on the nature of the game, it is possible that there exists a *second movers advantage* that the Centre wishes to exploit. The basic assumption is Centre acting as a dictator, writes the fiscal constitution in order to maximize federal welfare. If it happens that the Centre maximizes its welfare by being a second mover in the grant dispensation game, constitution may have to have a provision for soft budgets and bailouts.

To formalize the trade-off facing a central government, we assume that Centre has a fixed fund which finances a central project as well as transfer to provinces. Transfers are earmarked for provincial projects. There also exist two way "spillover" between provincial and central public good, that is, each level of government value outcome of the project at other sphere. Under this set up, we show that there is a possibility of second movers' advantage for the central government. The result depends on the degree to which central project benefits provinces. If the benefit is high (i.e. the central project produces a national public good), then Centre is better off by being a second mover. On the other hand, if the link between central project and provincial welfare is small or non-existent, the Center is better off being a first mover. As a policy prescription, our suggestion is that there is little room for "one-size-fits-all" type of transfer protocol. Rather, such transfer protocols are to be based on the nature of central project. We also show that, when centre chooses to be the second mover, under certain conditions, it can induce provinces to generate higher level of local revenue. These results run contrary to prevailing Hamiltonian wisdom.

It is to be noted that other authors (e.g. Besfamille and Lockwood, 2008) have already explored the relative inefficiency of hard budgets (Centre as first mover) vis-à-vis soft budget (mostly ex post grants) constraints in a federal set up. A key tool for their analysis is imperfect information and the associated moral hazard problem. Provincial revenue raising activities are not observable by the central authority. In real life, however, due to availability of budgetary documents in public domain and continuous scrutiny by media, such an assumption is hard to maintain.[1] Koethenbuerger (2008) also demonstrates possibility of welfare improvement when either Centre or provinces can pre-commit: this is achieved by putting a brake to the ‘race to the bottom'. Silva (2015) considers the regime of earmarked grants (central transfers which are tied to a specific public project) show that such grants may improve overall efficiency if provinces have the ability to commit (as a first mover in a sequential game). In this strand of literature, however, the endogeneity of transfer protocols is not explained.

Given that the present analysis focuses on the first mover and second mover advantages, the research contribution also spills over to the timing game paradigm in a federal set up. Timing games have been well researched in Industrial Organization literature (e.g. Gale-Or 1985; Dowrick 1986; Hamilton and Slustky 1990; Aamir and Stepanova 2006). The central research problem of timing games is to figure out conditions under which a leader or follower in a von-Stackelberg game (usually, a duopoly) is identified endogenously. The methodology was first used in context of a federation by Kemph and Graziosi (2010). In this paper, the authors address the issue of leadership between countries with transboundary externalities (e.g. environmental externalities) in a perfect information set up. However, the emphasis is on the interaction between countries, not between hierarchical governments, which is a feature of federal economies.

In sum, there are two different strands of literature within fiscal federalism regarding commitment. One preoccupies itself with the consequences of different commitment protocols within a federation with hierarchical governments, but does not explain how the protocols arise. The second strand discusses the origin of such protocols under different contexts, but does not include the hierarchical framework typical in a federation. The present study attempts to build a bridge between these two strands: by providing an explanation of first and second mover advantages in a federation with central and provincial governments.

The current paper (like Kemp and Graziosi, *ibid*) uses the taxonomy of strategic variables provided by Eaton (2004).[2] Eaton shows that second mover advantage in a general duopoly game is present only when both players have upward sloping reaction function. However, one important difference is the following. In his treatment, Eaton has assumed players with symmetric payoffs. Since the players in the current analysis are province and Centre, the payoff of each agent is asymmetric in nature. In the present paper, a second movers' advantage can be detected even if the players have downward sloping reaction function.

[1] That is, provinces are too big to escape notice of media or the central government etc.
[2] See appendix A.

2. Model

We assume a simple federal set up consisting of Centre and two provinces $(i=1,2)$. Provinces derive utility from a local project (outcome p_i) and local consumption $(\mathrm{c_i}, \text{which equals net income after taxes})$. Central authority gets benefit from a central project (outcome P). In addition, both central and provincial projects are valued by other tiers of government. Centre faces a budget constraint: $P+T_1+T_2=M$, where T_i is the transfer made to province $i.M$, the total amount of central fund, is exogenous.

Provincial welfare is

$$\mathrm{w^i(p_i, c_i, P) = u(p_i) + v(c_i) + \beta f(P)} \quad \text{..........(1)}$$

$f(.)$ represents provincial benefit from central project. The parameter β is the weight (or, equivalently, a parameter that captures marginal/total benefit) that the province puts on benefit from central project. Different values of β and different forms of f allow us to capture many facets of reality.[3] For example, if $\mathrm{f(.)} \equiv \mathrm{P}$ and $\beta=1$, then the centrally produced good assumes the nature of a national public good (e.g. a lighthouse) within the federation.

Similarly, central welfare depends on p_i and P :

$$\mathrm{W(p_1, p_2, P) = V(P) + \gamma(F(p_1) + F(p_2))} \quad \text{..........(2)}$$

Here, V is the benefit that Centre receives from own project. $\mathrm{F(.)}$ is the benefit that the Centre receives from provincial project and γ is the weight on associated benefits (equivalently, a parameter characterizing marginal benefit).[4] Again, this formulation allows us to capture many facets of reality. To focus on the hierarchical behavior of Centre and provinces, we assume away inter-provincial benefit from p_i. We also make the familiar assumptions: $\mathrm{u', v', V', f', F'} > 0$ and

[3]If P is armament import, provinces may perceive that the benefit is close to 0. If P is federally sponsored road network, β could be quite high. There is no upper limit on β .

[4]Traditionally, in Fiscal Federalism literature, Centre is seen as a Benevolent dictator which has sum of provincial utilities as objective function. However, such formulation may not address Hamilton's fear that reckless spending by provinces will squeeze central fund. The extent to which Central government cares for provinces is captured by the $\gamma \sum F_i(.)$ term. It may be noted that such non-Benthamite formulation of Federal welfare is not without precedence, e.g. see Snowdon and Wen (2003). In their formulation, provincial cost reduces Centres' welfare. In our formulation, provincial project outcome increases central benefit.

$u'', v'', V'', f'', F'' < 0$. As mentioned in section 1, a further assumption is that the Center has an overriding presence, such that its best interest is will be protected in the constitution and be institutionalized.

We assume that provinces are identical (save in income endowment, y_i). We need this assumption in order to induce identical transfer protocols for both provinces, i.e. if Centre is leader (follower) with respect to one province, it cannot behave differently towards other province.

3. Characterizing Equilibrium Protocols

Now we put more structure to the model by explicitly bringing in the nature of central grants. Grants are conditional in the sense that these are tied to a specific provincial project. Province raises a tax, say θ_i , to finance the public good project. Centre provides the transfer T_i to province i such that $T_1 + T_2 + P = M$. Provincial public good is $p_i = \theta_i + T_i$. Provincial consumption is $c_i = y_i - \theta_i$.[5] Incorporating these information in the utility functions, we can write equations (1) and (2) as functions of θ_i and T_i 's, where $i = 1, 2$

3.1 Reaction Functions

Province i chooses θ_i to maximize[6]

$$w(\theta_i, T_1, T_2) = u(\theta_i + T_i) + v(y_i - \theta_i) + \beta f(M - T_1 - T_2) \qquad (3)$$

From the first order condition,

$$u'(\theta_i + T_i) = v'(y_i - \theta_i) \qquad (4)$$

The slope of provincial reaction function is

$$\theta'(T_i) = -\frac{u''}{u'' + v''} < 0 \qquad (5)$$

[5]It is possible that central fund depends on federal income tax, i.e. $M = \tau(y_1 + y_2)$, where τ is the rate of tax and $c_i = (1 - \tau)y_i - \theta_i$. This will not change our results, as long as τ is exogenously given. We assume away from the issue.

[6]It can be shown that the associated Hessian matrix is negative definite (irrespective of the value of β). Thus the problem of non convexity does not arise.

Thus, $\mathrm{T_i}$ is strategic substitute for θ_i. Since raising θ_i is costly for the province, higher $\mathrm{T_i}$ will reduce θ_i. Note that $\frac{\partial p_i}{\partial T_i} = \frac{v''}{u''+v''} > 0$ on the reaction function of the province.

Similarly, Centre chooses T_1, T_2 to maximize[7]

$$\mathrm{W(T_1, T_2, \theta_1, \theta_2) = V(M - T_1 - T_2) + \gamma[F(\theta_1 + T_1) + F(\theta_2 + T_2)]} \quad \text{……….(6)}$$

The first order conditions are, for $i = 1, 2$

$$-\mathrm{V'(M - T_1 - T_2) + \gamma F'(\theta_i + T_i) = 0} \quad \text{………………………………………...(7)}$$

From the first order conditions, we can express the reaction function of Centre as $\mathrm{T_i(\theta_1, \theta_2)}$. It can be shown that

$$\begin{aligned} \frac{\partial \mathrm{T_i}}{\partial \theta_i} &= -\frac{\gamma \mathrm{F''(V'' + \gamma F'')}}{\mathrm{(V'' + \gamma F'')^2 - (V'')^2}} < 0 \\ \frac{\partial \mathrm{T_j}}{\partial \theta_i} &= \frac{\gamma \mathrm{F''V''}}{\mathrm{(V'' + \gamma F'')^2 - (V'')^2}} > 0 \end{aligned} \quad \text{………………………………………………………..(8)}$$

Similarly for θ_j. Thus, an increase in θ_i will reduce T_i but increase T_j. Notice that, following an increase in θ_i, the sum $(\mathrm{T_1 + T_2})$ falls, and hence an increase in provincial taxation reduces *total* central transfer and *quid pro quo*, increases output from central project on the reaction function of Centre.

The slope of central reaction function is *negative* on $\mathrm{T_i - \theta_i}$ plane.[8] The reason is as follows. Higher T_i is costly for the Centre (as its own public good production decreases). At the same time, with higher provincial taxation, provincial welfare from public good will increase. Since Centre cares for the provincial public good, $(\gamma \neq 0)$ Centre's response is to reduce the transfer as θ_i increases.

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[7]We cannot assume the benefits from provinces to be $F(p_1 + p_2) = F(\sum(\theta_i + T_i))$ since that would leave transfer to individual provinces (T_i) indeterminate.

[8]Higher θ_j will 'blow' the reaction function to right.

Example 3.1 *Let* $u(.) = p_i - \frac{\delta}{2} p_i^2$, $v(.) = c_i - \frac{\rho}{2} c_i^2$, $f(.) = P - \frac{\lambda}{2} P^2$, $V(.) = P - \frac{\eta}{2} P^2$;

$F(.) = p_i - \frac{\varphi}{2} p_i^2$ *. We assume that the parameters* $\delta, \rho, \lambda, \eta, \varphi$ *are small enough to always guarantee positive marginal utility.*

For province i *, the FOC is*

$$1 - \delta(\theta_i + T_i) = 1 - \rho(y_i - \theta_i)$$
$$\Rightarrow \theta_i(T_i) = \frac{\rho y_i - \delta T_i}{\delta + \rho}$$

For Centre, the FOC's are

$$-1 + \eta(M - T_1 - T_2) + \gamma(1 - \varphi(\theta_1 + T_1)) = 0$$
$$-1 + \eta(M - T_1 - T_2) + \gamma(1 - \varphi(\theta_2 + T_2)) = 0$$

Solving this, we get

$$T_i(\theta_i, \theta_j) = \frac{\gamma + M\eta - 1}{2\eta + \gamma\varphi} - \frac{\eta + \gamma\varphi}{2\eta + \gamma\varphi}\theta_i + \frac{\eta}{2\eta + \gamma\varphi}\theta_j$$

On central reaction function, total transfer is $T_1 + T_2 = \frac{2(\gamma + M\eta - 1)}{2\eta + \gamma\varphi} - \frac{\gamma\varphi}{2\eta + \gamma\varphi}(\theta_1 + \theta_2)$

Outcome of central project $P = \frac{M\gamma\phi + 2(1 - \gamma)}{2\eta + \gamma\varphi} + \frac{\gamma\varphi}{2\eta + \gamma\varphi}(\theta_1 + \theta_2)$

The structure of the problem allows us to treat interaction between different provinces and Centre separately. For example, the reaction functions of Centre and province i can be plotted in the $\theta_i - T_i$ plane, keeping θ_j as a parameter, which is determined in the $\theta_j - T_j$ plane.

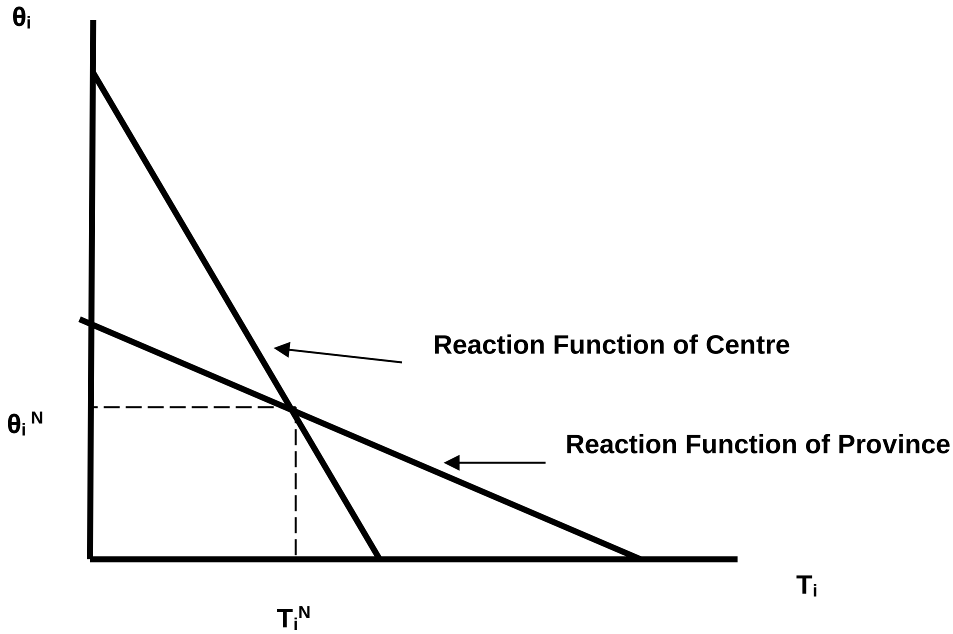
3.2 Nash Protocol

The Nash outcome $(\theta_1^N, \theta_2^N, T_1^N, T_2^N)$ is solution of the following equations:

$$\frac{\partial w(\theta_i^N, T_1^N, T_2^N)}{\partial \theta_i} = 0; i = 1, 2$$
$$\frac{\partial W(T_1^N, T_2^N, \theta_1^N, \theta_2^N)}{\partial T_i} = 0; i = 1, 2 \qquad \text{...(9)}$$

As it is clearly demonstrated, we have a Cournot type game with downward sloping reaction functions.

Figure 1: Nash Equilibrium



It can be shown that the equilibrium is stable. A Nash outcome is more likely if neither provinces, nor Centre are able to commit or reach a binding, enforceable constitution.

3.3 Stackelberg Protocols(s)

Here, we define the problems first.

If Centre is the first mover, then it chooses T_1 and T_2 in such a way that

$$W(T_1, T_2) = V(M - T_1 - T_2) + \gamma\left[F(\theta_1(T_1) + T_1) + F(\theta_2(T_2) + T_2)\right]$$

is a maximum. Here, $\theta_i(T_i)$ is the reaction function of the province (obtained from equation 4).

In symbols, the first order condition at the optimum can be written as, for $i = 1, 2$

$$\frac{\partial W(\theta_1^F,\theta_2^F,T_1^L,T_2^L)}{\partial T_i}+\frac{\partial W(\theta_1^F,\theta_2^F,T_1^L,T_2^L)}{\partial \theta_i}*\frac{\partial \theta_i}{\partial T_i}=0 \text{.......................(10)}$$

Solution to two equations will yield T_i^L . Plugging into provincial reaction functions, we get θ_i^F .

We show how γ alters T_i . In order to demonstrate the result, we continue with the LQ example.[9]

Lemma 3.1: *If* γ *increases, then* T_i^L *increases. As a result,* P^L *falls and increases.*

Proof: See appendix A2.

If γ increases, then Centre places lower weight on its own project and higher weight on provincial projects. Hence, transfers increase.

If province i (given the symmetry of provinces, similar conditions can be derived for province j) is leader vis-a-vis the Centre, then it has to choose θ_i in such a way that

$$w^i = u(\theta_i + T_i(\theta_1,\theta_2)) + v(y-\theta_i) + \beta f(M - T_1(\theta_1,\theta_2) - T_2(\theta_1,\theta_2))$$

is maximum. Here, $T_i(\theta_i,\theta_j)$ is the reaction function of the Centre (obtained from equation 7). In symbols, the first order condition can be written as, for $i=1,2$

$$\frac{\partial w^i(\theta_1^L,\theta_2^L,T_1^F,T_2^F)}{\partial \theta_i}+\sum_{j=1,2}\left[\frac{\partial w(\theta_1^L,\theta_2^L,T_1^F,T_2^F)}{\partial T_j}*\frac{\partial T_j}{\partial \theta_i}\right]=0 \text{.....................(11)}$$

Solutions to above equations will yield $\theta_i = \Theta^i(\theta_j)$ and $\theta_j = \Theta^j(\theta_i)$. Since our main concern is hierarchical structure *between the Centre and provinces*, we refrain from inter-provincial commitment issues that may arise when provinces are first mover. That is, (θ_1^L,θ_2^L) is determined by a simultaneous move game. Continuing with the LQ example, we prove a couple of lemmas to characterize provincial interaction. We focus on the parameter β because it takes a

[9] If we do not assume LQ functional form, then, in the comparative statics analysis, second derivatives of the reaction function such as $\frac{\partial^2\theta_i}{\partial T_i^2}$ etc. involve third derivatives of the utility functions. This becomes difficult to interpret.

central stage in our model.

Lemma 3.2: *If β is sufficiently high, then θ_1, θ_2 are strategic substitutes in the simultaneous move game.*

Proof: See appendix A2.

Note that, if $f(.) = M - T_1 - T_2$ so that the second derivative is zero, then θ_1 and θ_2 are always strategic complements.

Lemma 3.3: *If β increases, then equilibrium θ_i^L increases.*

Proof: See appendix A2.

It can be shown that the partial effect of β on θ_i, $\left(\frac{\partial\theta_i}{\partial\beta}\right)$ is positive. In case of strategic complementarity between θ_1 and θ_2, the total effect of β on θ_i (consisting of the partial effect as well as the indirect effect via θ_j) is unambiguously positive. In case of strategic substitutability between θ_i and θ_j, the total effect is potentially ambiguous. However, as the above lemma proves, the effect is also positive.

4. Determination of Outcomes

To determine the first and/or the second movers' advantage in the Stackelberg game, we need to figure out the shape of iso-welfare curves of the province as well as the Centre. That is, we need to determine whether the strategic variables chosen by different tiers of government are plain substitutes/complements.

4.1 Provincial Iso-welfare Curves

Let us first look at the provinces. The iso-welfare curve is defined by

$$\overline{w}_i = u(\theta_i + T_i) + v(y_i - \theta_i) + \beta f(M - T_1 - T_2) \quad \text{..12}$$

The slope of the curve in the $\theta_i - T_i$ plane[10] is $\frac{dT}{d\theta}|_{\overline{w}} = -\frac{w_\theta}{w_T}$ and $\frac{d^2T}{d\theta^2}|_{\overline{w}} = -\frac{w_{\theta\theta}}{w_T}$ *on the reaction function of province*. Therefore, $\text{sign}\left(\frac{d^2T}{d\theta^2}|_{\overline{w}}\right) = \text{sign}(w_T)$ at the critical point. Now $w_T = u'(p) - \beta f'(P)$. Theoretically, higher central transfer leads to higher output from provincial project, but the amount spent on central output reduces *quid-pro-quo*, and hence there is loss of provincial utility. For low values of $\beta(\approx 0)$, higher T is likely to be associated with higher provincial welfare. Hence T is plain complement (PC)[11] for province and the iso-welfare curves achieve a *minimum* on the reaction function. On the other hand, if β is sufficiently high, then the provincial iso-welfare curves achieve a maximum on the reaction functions, and higher T is associated with lower provincial welfare.

Example 4.1 *Let us illustrate the point using the LQ specification. We have* $w_T = 1 - \delta p - \beta(1 - \lambda P)$ *. Suppose* $\lambda = 0$ *. Then,* T_i *will always be a plain substitute (resp. complement) for province* i *if* β *is close to* $1\ (\text{resp.} 0)$ *.*

4.2 Central Iso-welfare Curves

The Central Iso Welfare curve in $\theta_i - T_i$ plane is defined by the following equation

$$\overline{W} = V(M - T_1 - T_2) + \gamma\left[F(\theta_1 + T_1) + F(\theta_2 + T_2)\right]$$

Following logic similar to above subsection, we find that, *on* the central reaction function facing province i, $\frac{d^2\theta}{dT^2}|_{\overline{W}} = -\frac{W_{TT}}{W_\theta}$, such that $\text{sign}\left(\frac{d^2\theta}{dT^2}|_{\overline{W}}\right) = \text{sign}(W_\theta)$. Now $W_\theta = \gamma F'() > 0.$ hence $W_\theta > 0$. *i.e.* the central iso-welfare curve always has a minimum on the reaction function of the Centre and higher θ implies higher welfare for Centre (provincial tax is always a plain complement for Centre).

4.3 Second Movers' Advantage

Now we are ready to state our main proposition.

[10]So as to avoid division by zero. We have also removed subscripts to retain notational clarity.
[11]See appendix A

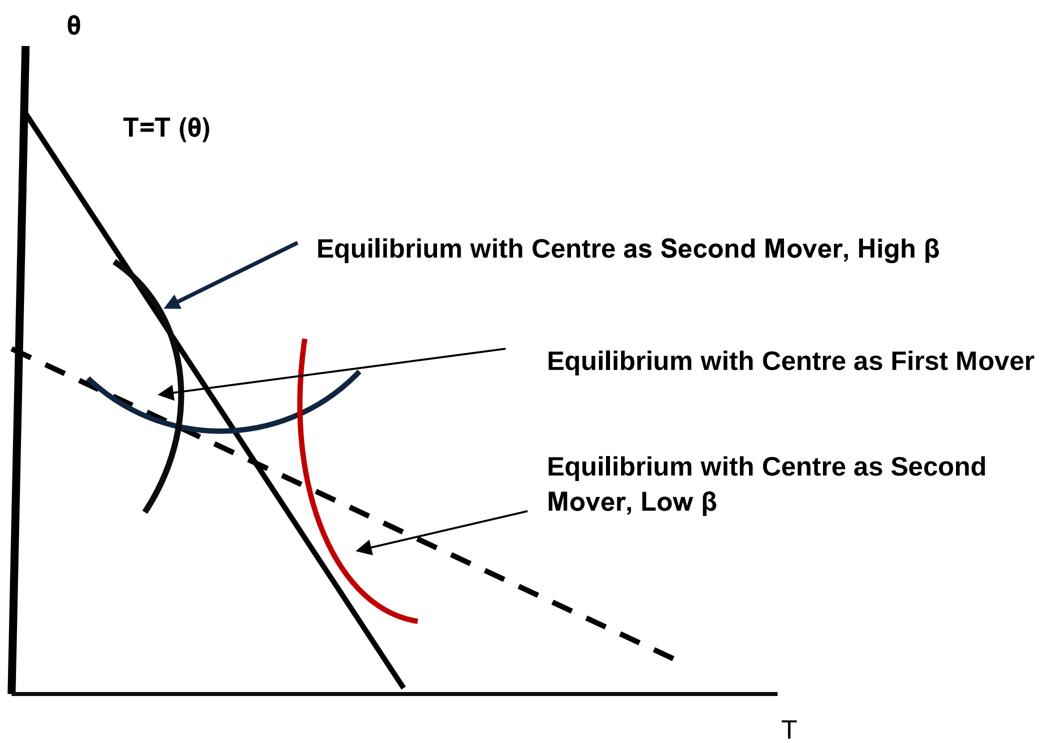
Proposition 4.1: *Assume that central grants are targeted to a specific provincial project. If the weight attached to central good by the provinces* (β) *is sufficiently high, then central transfer is plain substitute for the province. As a result, Centre is better off as a second mover. Otherwise, Centre is better off as a first mover.*

Proof: See appendix B.

Note that, given $W_\theta > 0$, Centre wishes to settle for θ as high as possible. It is evident that, if β is low, the provincial authority would like to have a high central transfer-low provincial taxes regime (because the cost of public good can be effectively shifted to the Centre and it does not depend on central public good so much). In that case, Centre gains by being the first mover and restrict the transfer.

This discussion can be summarized in the following diagram:

Figure 2: Centre Gains as Second Mover If $W_\theta > 0$ and $w_T < 0$



5. Comparison: Central and Provincial Leadership

Here we compare the two equilibria in terms of project outcomes and transfers. Assuming similar provinces, provincial taxes are identical and can be represented by a common notation θ.

Similarly, central transfer, in equilibrium, will be same for both provinces and can be denoted by a generic T. Let the three equilibria (Nash, and two Stackelberg points) be given by $\left\langle \theta^N, T^N \right\rangle$, $\left\langle \theta^F, T^L \right\rangle$ and $\left\langle \theta^L, T^F \right\rangle$, respectively. We now have the following proposition.

Proposition 5.1:

(i) *Assume* $w_T > 0$. *Then,* $\theta^F > \theta^N > \theta^L$ *and* $T^F > T^N > T^L$.

(ii) *If* β *is large enough, such that* $w_T < 0$. *then* $\theta^L > \theta^F > \theta^N$ *and* $T^F > T^L > T^N$ *with the LQ example.*

Proof: See appendix B.

As a corollary, we immediately have the following, for small β (when the constitutional outcome is Centre as first mover)

Corollary 5.1:

(ai) *Central transfers are lowest when Centre is a leader. That is, central project has highest output when Centre is leader:* $P^L > P^F$.

(aii) *Central leadership point is associated with highest provincial taxes, that is* $\theta^F > \theta^L$.

The effect on provincial public good, however, is ambiguous. As T decreases from T^F to T^N, θ increases along the central reaction function. Since the marginal response is more than 1, output of provincial project increases, $p^N > p^L$. As T further decreases from T^N to T^L, θ increases along the provincial reaction function. However, the marginal response is less than 1, which means lower provincial public good. The ultimate effect will depend on (given the slope of reaction functions), the relative distances between T^F, T^N and T^L .

The next corollary establishes the fact that with sufficiently high β ,

Corollary 5.2:

(bi) Central transfers are higher with Centre as second mover (that is $T^F > T^L$). As a result, central project yields lower output $(P^F < P^L)$ in such cases.

(bii) Provincial tax efforts are higher when provinces are leaders $\theta^L > \theta^F$. Thus $p^L > p^F$.

Of particular interest is the result (bii) of corollary 5.2. We know that, with sufficiently high β , Centre is a second mover in the game. Thus, both provincial revenue and outcome of provincial projects are higher when Centre *optimally* chooses not to pre-commit, vis-à-vis the case when Centre commits to the transfer.

6. Conclusion

In this paper, we have provided a general scenario under which the central government *gains* by limiting its power of commitment in the grant dispensation game within a federation. If central government grant is tied up with the public project of the province, provincial tax and central transfer are strategic substitutes. Higher central transfer lowers the marginal utility of public project to the province and province responds by cutting down taxes. Higher central transfer increases provincial welfare, but decreases output from central project. In case provinces attach high weight on central project, higher central transfer will reduce provincial welfare. In that case, provinces respond by increasing their own taxes. If central welfare is increasing in provincial taxes, then Centre should choose a mode of transfer which generates higher provincial taxes. Thus, contrary to the Hamiltonian paradigm, provincial taxes can be higher even if central government is unable to commit.

The work can be extended to several dimensions. First, we have demonstrated the result with one type of grant. In a federal economy, the grants may just augment provincial budget instead of being tied to a project. It would be interesting to extend the result to such grants. Second, a key assumption of the paper is Centre has an over-riding presence in dictating the mode of transfer. In many economies, the balance of power between Centre and provinces are determined by bargaining, e.g. during nascent years of the USA. This opens up the possibility of a timing game to resolve the tie of leader/follower. Third, we have assumed that β for each province is same. But suppose it is not: some provinces, due to political alignment with Centre, have a high β , other provinces perceive β to be quite low. Then Centre may be a first (second) mover with the latter (former) group of provinces: we would expect provincial debt services being assumed by the central government in the politically aligned provinces. This suggests the familiar spectre of partisan behavior. But such partisan effect does not arise from Centre putting different weight on provincial welfare, (as assumed in traditional political economy literature, e.g. Sengupta 2011), but from *provinces placing different weights on central project*.

Thus, there exists future scope of research based on the current work.

Appendix A
Strategic and Plain Complements

Briefly, suppose $\pi^i(a_i, a_j)$ is benefit function for agent i, while a_i and a_j are own actions and other agents' action, respectively. Then a_j is plain complement (PC) for agent i if $\frac{\partial \pi^i}{\partial a_j} > 0$, and plain substitutes (PS) if $\frac{\partial \pi^i}{\partial a_j} < 0$. Similarly, a_i and a_j are strategic substitutes, SS (complements, SC) if $\frac{\partial^2 \pi^i}{\partial a_i \partial a_j} < (>) 0$. Similarly for agent j.

The first order condition for agent i is

$$\frac{\partial \pi^i}{\partial a_i} = 0$$

Differentiating with respect to a_j, we have

$$\frac{\partial^2 \pi^i}{\partial a_i^2}\frac{da^i}{da_j} + \frac{\partial^2 \pi^i}{\partial a_i \partial a_j} = 0$$

$$\rightarrow \frac{da^i}{da_j} = -\frac{\frac{\partial^2 \pi^i}{\partial a_i \partial a_j}}{\frac{\partial^2 \pi^i}{\partial a_i^2}}$$

If SOC holds, the sign of cross (double) derivative determines the slope of reaction functions. In the same vein, sign of cross (single) derivative determine the shape of iso-profit curves near the reaction function.

Let the iso-profit curve for agent i in $a_i - a_j$ plane be $\pi^i(a_i, a_j) = \bar{k}$. The slope is

$$\frac{da_j}{da_i} = -\frac{\pi_i^i}{\pi_j^i}.$$

Here, the lower subscripts denote partial derivatives, i.e. $\frac{\partial \pi^i}{\partial a_i} = \pi_i^i$. On the reaction function. $\pi_i^i = 0$, so that is a critical point of the iso-profit (or iso-welfare) curve. Notice that

$$\frac{d^2 a_j}{da_i^2} = -\frac{\pi_{ii}^i\left(\pi_j^i\right)^2 + \pi_{jj}^i\left(\pi_i^i\right)^2 - 2\pi_{ij}^i\pi_i^i\pi_j^i}{\left(\pi_j^i\right)^3}$$

$$= -\frac{\pi_{ii}^i}{\pi_j^i} \text{ at } \pi_i^i = 0$$

Thus, $\operatorname{sign}\left(\frac{d^2 a_j}{da_i^2}\right) = \operatorname{sign}\left(\pi_j^i\right)$ at the critical point.

Appendix B
Proof of Lemma 3.1

From equation 10, we can write the Hessian matrix of second derivatives for Centre welfare

$$H = \begin{bmatrix} \frac{\partial^2 W}{\partial T_1^2} & \frac{\partial^2 W}{\partial T_1 \partial T_2} \\ \frac{\partial^2 W}{\partial T_1 \partial T_2} & \frac{\partial^2 W}{\partial T_2^2} \end{bmatrix}$$
$$= \begin{bmatrix} V'' + \gamma F'' k^2 & V'' \\ V'' & V'' + \gamma F'' k^2 \end{bmatrix}$$

Here $k = 1 + \frac{\partial \theta_i}{\partial T_i} = \frac{\rho}{\delta + \rho}$. Of course, $V'' + \gamma F'' k^2 < 0$ and $|H| = \left(V'' + \gamma F'' k^2\right)^2 - \left(V''\right)^2 > 0$.

Thus $W(.)$ is concave.

Differentiating the FOC with respect to γ , we get the following matrix equation

$$\begin{bmatrix} V'' + \gamma F'' k^2 & V'' \\ V'' & V'' + \gamma F'' k^2 \end{bmatrix} \begin{bmatrix} \frac{dT_1}{d\gamma} \\ \frac{dT_2}{d\gamma} \end{bmatrix} = \begin{bmatrix} -kF' \\ -kF' \end{bmatrix}$$

$$\Rightarrow \frac{dT_i^L}{d\gamma} = -\frac{kF'(\gamma\gamma^2 F'')}{|H|} > 0 \text{ for } i = 1, 2$$

Thus $\frac{dP^L}{d\gamma} = -\frac{d\left(\sum T_i\right)}{d\gamma} < 0$, $\frac{d\theta_i^F}{d\gamma} = \theta'\left(\right)\frac{dT_i}{d\gamma} < 0$ and $\frac{dp_i^F}{d\gamma} = \left(1 + \theta'(T_i)\right)\frac{dT_i^L}{d\gamma} > 0$.

Proof of Lemma 3.2

Equation (11) can be written as, say for province 1:

$$\frac{\partial w^1}{\partial \theta_1} = u'(\theta_1 + T_1)\left(1 + \frac{\partial T_1}{\partial \theta_1}\right) - v'\left(y_1 - \theta_1\right) + u'(\theta_1 + T_1)\frac{\partial T_1}{\partial \theta_1} - \beta f'(P)\left(\frac{\partial T_1}{\partial \theta_1} + \frac{\partial T_2}{\partial \theta_1}\right) = 0$$

Assuming LQ functional forms, $\frac{\partial T_1}{\partial \theta_1}$ and $\frac{\partial T_2}{\partial \theta_1}$ are constant numbers. Differentiating above expression with respect to θ_2 , we get

$$\frac{\partial^2 w^1}{\partial \theta_1 \partial \theta_2} = u'' \frac{\partial T_1}{\partial \theta_2}\left(1 + 2\frac{\partial T_1}{\partial \theta_1}\right) + \beta f''(P)\left(\frac{\partial T_1}{\partial \theta_1} + \frac{\partial T_2}{\partial \theta_1}\right)\left(\frac{\partial T_1}{\partial \theta_2} + \frac{\partial T_2}{\partial \theta_2}\right)$$

Notice that the first term is positive, since $u'' = -\delta < 0$, $\frac{\partial T_1}{\partial \theta_2} = \frac{\eta}{2\eta+\gamma\varphi} > 0$ and $1 + 2\frac{\partial T_1}{\partial \theta_1} = 1 - 2\frac{\eta+\gamma\varphi}{2\eta+\gamma\varphi} = -\frac{\gamma\varphi}{2\eta+\gamma\varphi} < 0$. On the other hand, $f'' = -\lambda < 0$ and sum of $T_1 + T_2$ falls as θ_1 or θ_2 increases. So the second term is definitely negative. If β is large enough, then, even if the first term is positive, the whole expression $\frac{\varphi\gamma}{(2\eta+\gamma\varphi)^2}(\delta\eta - \beta\lambda\gamma\varphi)$ is negative. Hence, θ_1, θ_2 are strategic substitute since $\frac{\partial \theta_1}{\partial \theta_2} = -\frac{\left(\frac{\partial^2 w^1}{\partial \theta_1 \partial \theta_2}\right)}{SOC} < 0$.[12]

Proof of Lemma 3.3

To show this, we proceed in two steps. First, we show the (partial) effect of β on θ_i^L . Second, we demonstrate the total effect of β on θ_i^L (consisting of the direct effect as well as the indirect effect via θ_j^L .

In order to do so, we write the reaction functions (implicitly defined by equation 11) as $\theta_i = \Theta^i(\theta_j; \beta)$. Notice that, we have already demonstrated the fact that (lemma 3.1), if β is above a threshold then $-1 < \frac{\partial \Theta^i}{\partial \theta_j} < 0$.

To find the direct effect of β on θ_i, we need to find $\frac{\partial \Theta^i}{\partial \beta} = -\frac{\left(\frac{\partial^2 w^1}{\partial \theta_1 \partial \beta}\right)}{SOC}$. Given our assumptions, $\frac{\partial^2 w^1}{\partial \theta_1 \partial \beta} = -f'(P)\left(-\frac{\gamma\varphi}{2\eta + \gamma\varphi}\right) > 0$, so an increase in β increases θ_i , if nothing else changes.

To obtain the total effect (because of the ambiguity that higher β increases both θ_i and θ_j , but higher θ_j reduces θ_i) one has to differentiate the equations $\theta_i = \Theta^i(\theta_j; \beta)$ and

[12]The SOC is $\frac{\partial^2 w_1}{\partial \theta_1^2} = -\frac{\delta\eta^2}{(2\eta+\gamma\varphi)^2} - \rho - \beta\lambda\left(\frac{\gamma\varphi}{2\eta+\gamma\varphi}\right)^2 < 0$. It can be shown that the magnitude of the slope of the reaction function in $\theta_i - \theta_j$ space is less than one (Similarly for the other province. That is, the Nash equilibrium is stable.

$\theta_j = \Theta^j(\theta_i;\beta)$ and obtain $\frac{d\theta_i}{d\beta}$ etc. The resulting matrix equation is

$$\begin{bmatrix} 1 & -\frac{\partial\Theta^i}{\partial\theta_j} \\ -\frac{\partial\Theta^j}{\partial\theta_i} & 1 \end{bmatrix}\begin{bmatrix} \frac{d\theta_i}{d\beta} \\ \frac{d\theta_j}{d\beta} \end{bmatrix} = \begin{bmatrix} \frac{\partial\Theta^i}{\partial\beta} \\ \frac{\partial\Theta^j}{\partial\beta} \end{bmatrix}$$

Here, $$\Delta = \begin{vmatrix} 1 & -\frac{\partial\Theta^i}{\partial\theta_j} \\ -\frac{\partial\Theta^j}{\partial\theta_i} & 1 \end{vmatrix} = 1 - \left(\frac{\partial\Theta^i}{\partial\theta_j}\right)\left(\frac{\partial\Theta^j}{\partial\theta_i}\right) > 0$$

Thus,

$$\frac{d\theta^i}{d\beta} = \frac{\begin{vmatrix} \frac{\partial\Theta^i}{\partial\beta} & -\frac{\partial\Theta^i}{\partial\theta_j} \\ \frac{\partial\Theta^j}{\partial\beta} & 1 \end{vmatrix}}{\Delta} = \frac{\frac{\partial\Theta^i}{\partial\beta} + \frac{\partial\Theta^j}{\partial\beta}\frac{\partial\Theta^i}{\partial\theta_j}}{\Delta}$$

If we have LQ example (where provinces only differ by income level), $\frac{\partial\Theta^i}{\partial\beta} = \frac{\partial\Theta^j}{\partial\beta}$. So the numerator is $\frac{\partial\Theta^i}{\partial\beta}\left(1 + \frac{\partial\Theta^i}{\partial\theta_j}\right) > 0 \rightarrow \frac{d\theta_i^L}{d\beta} > 0$. Similarly for $\frac{d\theta_j^L}{d\beta}$

Thus, $\frac{d(\sum T_i^F)}{d\beta} = -\left(\frac{\gamma\varphi}{2\eta+\gamma\varphi}\right)\frac{d\left(\sum\theta_i^L\right)}{d\beta} < 0 \rightarrow \frac{dP^F}{d\beta} = -\frac{d(\sum T_i^F)}{d\beta} > 0$ and $\frac{dp_i^L}{d\beta} = \frac{d\theta_i^L}{d\beta} - \left(\frac{\eta+\gamma\varphi}{2\eta+\gamma\varphi}\right)\frac{d\theta_i^L}{d\beta} + \left(\frac{\eta}{2\eta+\gamma\varphi}\right)\frac{d\theta_j^L}{d\beta}$. Since $\frac{d\theta_j^L}{d\beta} \equiv \frac{d\theta_i^L}{d\beta}$ we have $\frac{dp_i^L}{d\beta} = 2(2\eta+\gamma\varphi)^{-1}\eta\left(\frac{d\theta^L}{d\beta}\right) > 0$. Thus transfers go down, but local taxation rises to compensate such that local project outputs increase.

Proof of Proposition 4.1

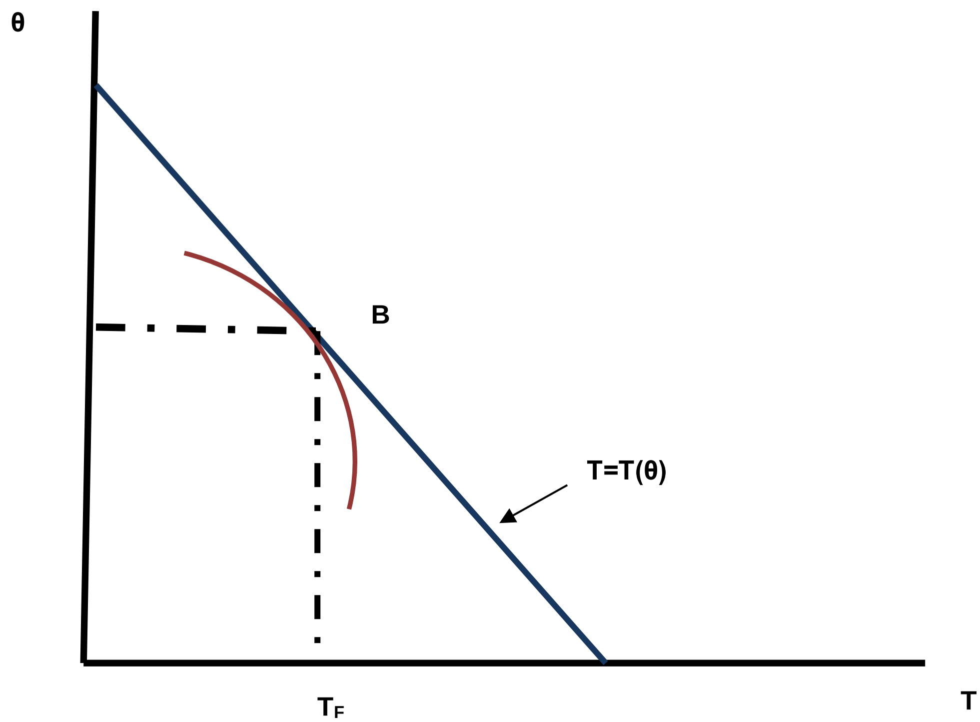
In this and subsequent proofs, we are omitting other variables (θ_j, T_j) for notational clarity. Also, the subscripts are dropped: that is $w^i(\theta_i^E, T_i^E \mid T_j^E, \theta_j^E) \equiv w^i(\theta_i, T_i^E)$ [13] $\equiv w(\theta^E, T^E)$ (subscripts are dropped). Here, the superscript E stands for different equilibria, e.g. Nash or Stackelberg. Similarly for the Centre.

[13] T_j being a parameter in the equilibrium

Second part of the proposition is evident: so only the first part is proved. The proof[14] follows Dowrick (1986). Notice that, we need to show that the Centre is better off as a Stackelberg follower than leader.

Proof: The proof proceeds in two stages. In the following figure, we have drawn reaction function of the Centre and a Stackelberg point B (θ_L, T_F) such that province is the leader (the iso-welfare curve is not necessary).

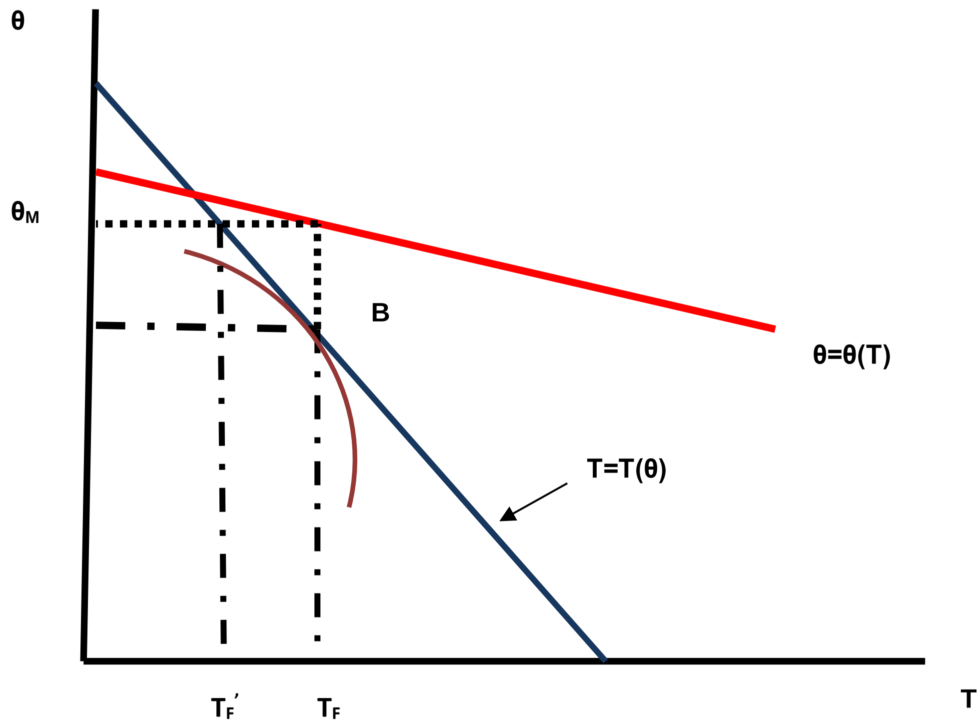
Figure A1: A Stackelberg Leader Point for Province



First, we show that the reaction function of the province $\theta = \theta(T)$ must be below B, i.e. $\theta(T_F) < \theta_L$.

[14] The proof does not depend on linear quadratic assumption.

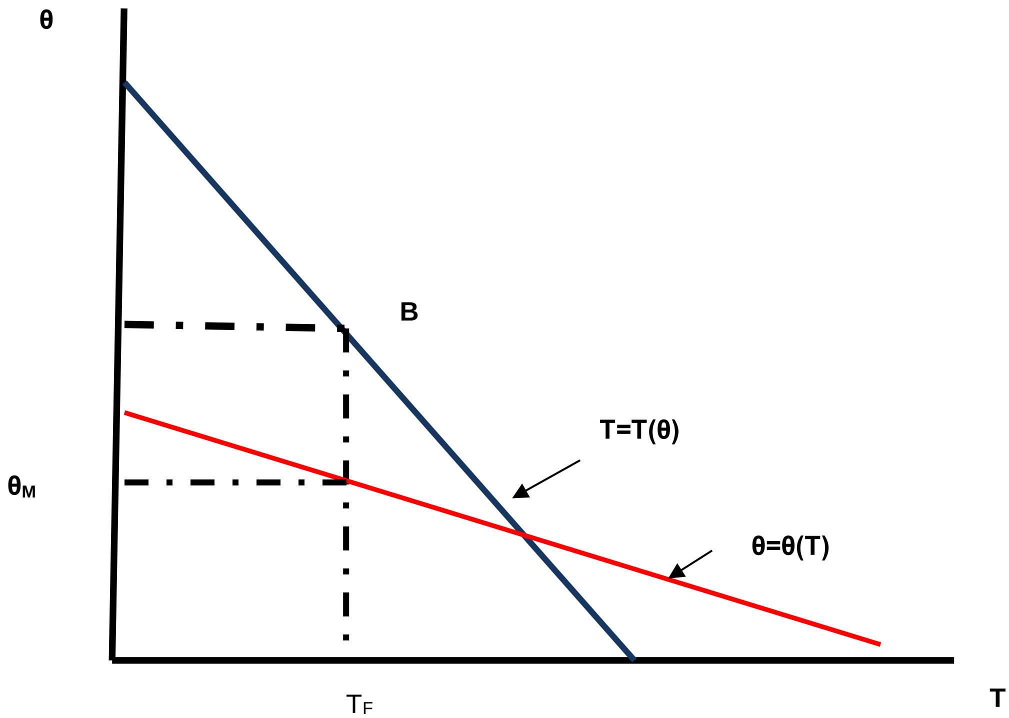
Figure A2: Position of the Provincial Reaction Function



Suppose not. Then $\theta_M = \theta(T_F) > \theta_L$. Thus we have, for the province, $w(\theta_M, T_F) > w(\theta_L, T_F)$ (by the definition of reaction function. Let, $T_F^{❖}$ be the best response to θ_M , that is, $T_F^{'} = T(\theta_M)$. But, then since w_T 🖫0, we have $w(\theta_M, T_F) < w(\theta_M, T_F^{'})$. Combining these inequalities, we get $w(\theta_M, T_F^{'}) > w(\theta_m, T_F) > w(\theta_L, T_F)$. But then, (θ_L, T_F) cannot be the Stackelberg leadership point of the province on the reaction function of Centre.

Second, $\theta = \theta(T)$ cannot pass through the Stackelberg leadership point B. Notice that the iso-profit curve of the province must have zero slope on $\theta = \theta(T)$. For simultaneous tangency on $T = T(\theta)$ a critical point of $\theta = \theta(T)$, the reaction function $T = T(\theta)$ must be positively sloped. But this case is ruled out either. Thus, $\theta = \theta(T)$ must be below B, the Stackelberg leadership point of the province. So, $\theta_M < \theta_L$.

Figure A3: Centre Prefers to Be Follower



Note that, $W(\theta_L, T_F) > W(\theta_M, T_F)$, since $W_\theta > 0$. But, (θ_M, T_F) is one of the set of points which the Centre can choose as a Stackelberg leader. Therefore, Centre must prefer to be a Stackelberg follower if $W_\theta > 0$ and $w_T < 0$.

Proof of Proposition 5.1 (Part i)

As before, we are focusing on the interaction between one province and the corresponding transfer.

Proof: Comparing provincial Nash and Leadership position, we have

$$w(\theta^L, T^F(\theta_L)) \geq w(\theta^N, T^N) \qquad \text{(B1)}$$

by definition of Nash equilibrium and Stackelberg leadership.

When province is follower in a Stackelberg game or under Nash protocol, equilibrium occurs *on* provincial reaction function. Thus, we must have, by the definition of reaction function,

$$\frac{\partial w(\theta^N,T^N)}{\partial\theta}=\frac{\partial w(\theta^F,T^L)}{\partial\theta}=0 \qquad \text{(B2)}$$

Given $w_{T\theta}<0$

$$\theta^N>\theta^F \Leftrightarrow T^N<T^L \qquad \text{(B3)}$$

Again, by definition of Nash equilibrium:

$w(\theta^N,T^N)\geq w(\theta^L,T^N)$

Suppose $T^N>T^F$. Then

$w(\theta^N,T^N)\geq w(\theta^L,T^N)>w(\theta^L,T^F)$

The second inequality follows from the fact that $w_T>0$. But this contradicts with the definition of Stackelberg leadership. Therefore, we must have $T^N<T^F$. Notice that T^N and T^F are on the same (downward sloping) central reaction function. The $\theta^N>\theta^L$. Similarly, for the Centre ($W_{T\theta}<0$ and $W_\theta>0$), we must have $\theta^F>\theta^N$. But θ^N and θ^F are two points on the (downward sloping) provincial reaction function. So we must have $T^L<T^N$. Combining these observations, we have result 5.1 (i).

Proof of Proposition 5.1 (Part ii)

Proof For part (ii), notice that β is high, so that $w_T<0$. Applying the same methodology as in 5.1 (part i), we have $T^N>T^F$, i.e. $\theta^N<\theta^L$. At the same time, $\theta^F>\theta^N$ i.e. $T^L<T^N$. That is, we have $(\theta^F,\theta^L)>\theta^N$ and $(T^F,T^L)<T^N$. However, the relative magnitudes of various leadership positions with the follower position is not known.

Let us assume $T^L>T^F$.

We compare the first order conditions of the equilibrium of the Centre. We already know that, when Centre is the follower, as well as in Nash outcome

$$\frac{\partial W(\theta^L, T^F)}{\partial T} = \frac{\partial W(\theta^N, T^N)}{\partial T} = 0$$

We compare this with the case when Centre is the leader. It solves the problem

$$\max_T W(T, \theta(T)) = 0$$

The FOC yields

$$\frac{\partial W(\theta^F, T^L)}{\partial T} + W_\theta * \frac{\partial \theta}{\partial T} = 0$$

Note that $\frac{\partial \theta}{\partial T} = -\frac{w_{\theta T}}{w_{\theta\theta}} < 0$. Given $W_\theta > 0$, we must have $\frac{\partial W(\theta^F, T^L)}{\partial T} > 0$.

Thus

$$\frac{\partial W(\theta^F, T^L)}{\partial T} > \frac{\partial W(\theta^L, T^F)}{\partial T} = \frac{\partial W(\theta^N, T^N)}{\partial T}$$

If $T^L > T^F$, then $\theta^F < \theta^L$.

Notice $\frac{\partial W(\theta^L, T^F)}{\partial T} > \frac{\partial W(\theta^L, T^L)}{\partial T}$ since MU falls with T and $T^L > T^F$.

Second $\frac{\partial W(\theta^L, T^L)}{\partial T} > \frac{\partial W(\theta^F, T^L)}{\partial T}$ since MU falls with θ $\left(W_{T\theta} < 0\right)$ and $\theta^L > \theta^F$.

Combining these two statements, $\frac{\partial W(\theta^L, T^F)}{\partial T} > \frac{\partial W(\theta^F, T^L)}{\partial T}$ which is a contradiction.

Thus $T^F > T^L > T^N$.

Note, however, this inequality does not tell us anything about relative values of θ^L and θ^F since these are on two separate reaction functions.

We have already noted

$$\frac{\partial w(\theta^F, T^L)}{\partial \theta} = \frac{\partial w(\theta^N, T^N)}{\partial \theta} = 0$$

When province i is leader, it maximises

$$\max_{\theta} w(\theta_i, T_i(\theta,.).T_j(.,\theta))$$

FOC is

$$w_\theta + \left[w_{T_i} * \frac{\partial T_i}{\partial \theta_i} + w_{T_j} * \frac{\partial T_j}{\partial \theta_i} \right] = 0$$

Notice that, $w_{T_i} < 0$ and $\frac{\partial T_i}{\partial \theta_i} < 0$. On the other hand, $w_{T_j} < 0$ and $\frac{\partial T_j}{\partial \theta_i} > 0$ (Note: $w_{T_i} = u' - \beta f'$ and $w_{T_j} = -\beta f')$. Using the LQ specification, $\frac{\partial T_i}{\partial \theta_i} = -\frac{\eta + \gamma\varphi}{2\eta + \gamma\varphi}$ and $\frac{\partial T_j}{\partial \theta_i} = \frac{\eta}{2\eta + \gamma\varphi}$.

Thus, the terms within square brackets can be written as

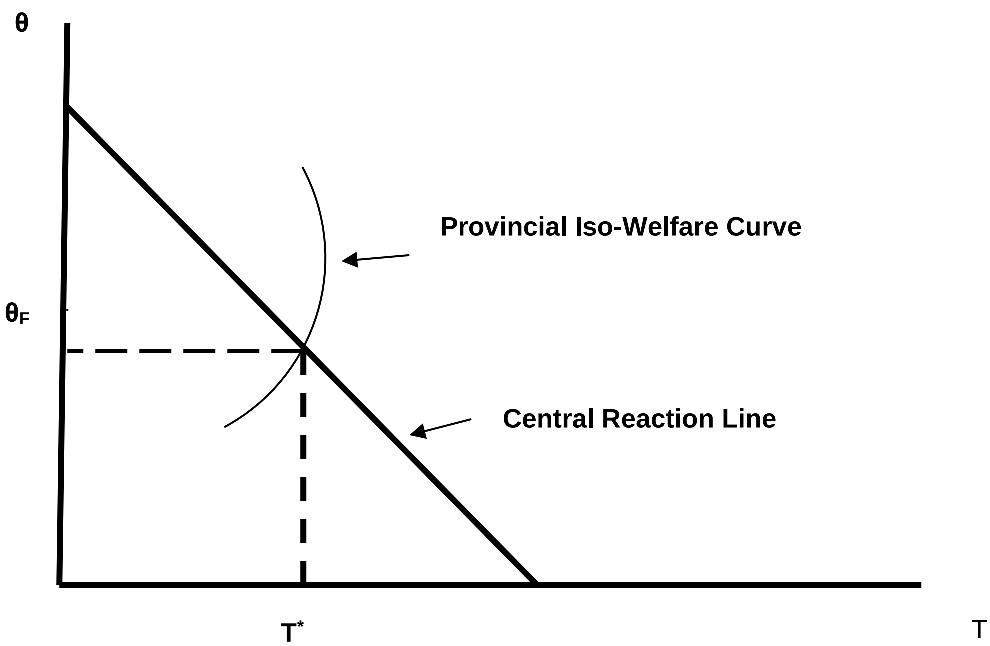
$$\begin{aligned}
& w_{T_1} * \frac{\partial T_1}{\partial \theta_1} + w_{T_2} * \frac{\partial T_2}{\partial \theta_1} \\
&= -(u' - \beta f')\frac{\eta + \gamma\varphi}{2\eta + \gamma\varphi} + \frac{\eta}{2\eta + \gamma\varphi}(-\beta f') \\
&= \beta f'\left(\frac{\eta}{2\eta + \gamma\varphi}\right) - u'\left(\frac{\eta + \gamma\varphi}{2\eta + \gamma\varphi}\right)
\end{aligned}$$

We already know that β is sufficiently large such that $\beta f' > u'$. We need to impose a mildly stringent restriction on β , i.e. $\beta f' > u'\left(1 + \frac{\gamma\varphi}{\eta}\right)$ such that expressions under square brackets is positive and

$$\frac{\partial w(\theta^L, T^F)}{\partial \theta} < \frac{\partial w(\theta^F, T^L)}{\partial \theta} = \frac{\partial w(\theta^N, T^N)}{\partial \theta}$$

A comparison between the first two terms imply that one cannot rank θ^L and θ^F from the condition stated above. However, if the iso-welfare curve of the province cuts the reaction function of the Centre at the point $\left(\theta^F, \tilde{T} = T(\theta^F)\right)$, then the tangency (which defines θ^L) between provincial iso-profit curve and central reaction function occurs at a *higher* point, that is $\theta^L > \theta^F$. The possibility is shown in the following diagram.

Figure A4 : Condition for θ^F > θ^L



Slope of the provincial iso-profit curve $\frac{d\theta}{dT}|_{(\theta^F, T^*)} = -\frac{u' - \beta f'}{u' - v'}$, while that of central reaction function in the $\theta_i - T_i$ plane is A_i (e.g. for LQ assumption, it is $\frac{2\eta+\gamma\varphi}{\eta+\gamma\varphi}$). So we need $\frac{u' - \beta f'}{u' - v'}|_{(\theta^F, T^*)} > A_i$. Since the point $\left(\theta^F, T^*\right)$ is above the reaction function of the province, we must have u❖✍v❖🖫0 . The inequality then suggests $\beta > \hat{\beta} = \frac{u' - A_i * (u' - v')}{f'}|_{(\theta^F, \tilde{T})}$ is required for $\theta^L > \theta^F > \theta^N$. This is also partially corroborated by lemma 3.3.

It is easy to (but tedious) figure out $\hat{\beta}$ for the LQ example. As we know,

$$T_i^L = \frac{M\eta - 1 + \alpha\gamma - (\eta + \alpha\gamma\varphi)y_i + \eta y_j}{2\eta + \alpha\gamma\varphi} \rightarrow \theta_i^F = \alpha y_i - (1-\alpha)T_i^L$$, where $\alpha = \frac{\rho}{\rho+\delta}$

This, in turn, implies that,

$$\begin{aligned} T_i^* &= \frac{\gamma + M\eta - 1}{2\eta + \gamma\varphi} - \frac{\eta + \gamma\varphi}{2\eta + \gamma\varphi}\theta_i^F + \frac{\eta}{2\eta + \gamma\varphi}\theta_j^F \\ \hat{p}_i &= \theta_i^F + T_i^* \\ \hat{P} &= \frac{M\gamma + 2(1-\gamma)}{2\eta + \gamma\varphi} + \frac{\gamma\varphi}{2\eta + \gamma\varphi}\left(\theta_1^F + \theta_2^F\right) \end{aligned}$$

From here, we can calculate $u' = 1 - \delta\hat{p}_i$, $v' = 1 - \rho(y_i - \theta_i^F)$ and $f' = -1 + \lambda(\hat{P})$.

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