ON LOCAL MEASURES OF TAX PROGRESSION: APPLICATIONS IN TAX DESIGN

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Abstract

The study introduces a new measure and explores applications of local measures in tax design. It suggests specific forms of tax design/structure which on equiproportional change in pre-tax incomes of all the taxpayers would result in desired effects on redistributive impact of the tax. Also it provides a technique to control/neutralise the resultant change, if any, in redistributive impact of the tax.

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1. Introduction

Progression in the income tax rate schedule implies departure from proportionality in the distribution of tax burden. It is characterised generally by an increasing average tax rate with income. There are several contemporary measures of tax progression which can be classified into two[1] broad categories, namely, local (also known as structural or scheduler) and global (also known as summary or distributional). A local measure constructs a schedule of tax rate or tax liability or post-tax income along the income scale. A global measure takes the form of a single number and it focuses, in general, on the distributional aspect of the tax in terms of tax liability or pre- and post-tax incomes.[2]

Traditionally the local measures of progression have been looked upon as "merely" different mathematical expressions of the relationship between income and tax (Pigou, 1947; Musgrave and Thin, 1948). Only recently these measures have evoked greater interest among researchers. These are now viewed as genesis of developing and testing the adequacy of even the global measures of tax progression and are indicative of intensity of the effects of progression (Aggarwal, 1980; Lambert, 1989, Chapter 7; Podder, 1990).

The focus of different studies relating to local measures of progression has remained either on compatibility of various measures of progression (Podder, 1990) or on association between the measures and effects of progression (Aggarwal, 1980; Lambert, 1989, Chapters 7 and 8). Aggarwal (1980), however, also draws attention towards applications of these measures in tax design, an area which has not been adequately explored. The main purpose of this study is to initiate the process of filling this gap. Also the study reviews salient features of contemporary local measures of tax progression and introduces a new measure that has applications in tax design.

The plan of the study is as follows. The salient features of various global measures of tax progressivity are given in Section 2. A new measure is introduced and its characteristics are studied in Section 3. A brief discussion on compatibility of these measures is contained in Section 4. Implications of the measures in tax design are discussed in Section 5. Conclusions are presented in section 6.

2. Salient Features of Local Measures of Progression

The contemporary local measures of tax progression are based either on the basic criterion that the average tax rate increases with income or on the assumed convexity of the rate schedule. These can be divided into four broad categories, namely, (i) those based on change in average tax rate with income, (ii) those based on change in marginal tax rate with income, (iii) those based on change in tax liability or post-tax income with income (pre-tax). The salient features of these measures are discussed below. A summary of these is given in Table 1. In general, conditions are stated and proofs are given for the case of tax progression. However, exactly analogous conditions can be stated and proofs given for the case of tax regression.

Let t(y) denote tax liability at income level y. For convenience, we define m(y) and a(y) respectively the marginal and average rates of tax at income level y as:

$$m(y) = t'(y) = \frac{dt(y)}{dy},$$

$$m'(y) = t''(y),$$

$$a(y) = t(y)/y, \text{ and}$$

$$a'(y) = \frac{d\,a(y)}{dy}$$

2.1 Measures of progression based on average tax rate

Musgrave and Thin (1948) proposed two measures of tax progression based on average tax rate, namely, average rate progression (ARP), and residual average rate progression (RARP) as the rate of change of average tax rate and residual average tax rate respectively. These differ only in sign and can be expressed as:

$$ARP(y) = \frac{da(y)}{dy} = \frac{m(y)-a(y)}{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$RARP(y) = \frac{d[1-a(y)]}{dy} = -ARP(y)$$

Both these measures depend on income level. Following Lambert (1989) these can be modified to make them neutral to income scale:

$$ARP^*(y) = y.ARP(y) = m(y) - a(y)$$

$$RARP^*(y) = \frac{y.\ d[1-a(y)]}{dy} = -ARP^*(y)$$

Aggarwal (1980) proposed a measure, namely, Average rate elasticity progression (AREP), proposed by Aggarwal (1980) is defined as the ratio of proportional change in average tax rate to the proportional change in income.

$$AREP(y) = \frac{da(y)/a(y)}{dy/y} = \frac{m(y) - a(y)}{a(y)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

ARP is invariant to additive[3] translations of average tax rates while AREP is invariant to proportional[4] translations of the average rates.

2.2 Measures of progression based on marginal tax rate

Based on changes in marginal tax rate with income, three measures of tax progression have been proposed. Marginal rate progression (MRP), proposed by Pigou (1947), is defined as the rate of change of marginal rate of tax with income:

$$MRP(y) = \frac{d\ m(y)}{dy} = m'(y) \qquad (3)$$

Marginal rate elasticity progression (MREP), proposed by Aggarwal (1980) and Lambert (1989) that captures concavity of the tax schedule is defined as the ratio of proportional change in marginal tax rate to proportional change in income:

$$MREP(y) = \frac{d\ m(y)}{m(y)} \Big/ \frac{dy}{y} \qquad (4)$$

Residual marginal rate elasticity progression (RMREP), proposed by Richter (1984), and Richter & Hampe (1984), that captures concavity of the post-tax income schedule is defined as the ratio of proportional change in residual marginal rate of tax to proportional change in income:

$$RMREP(y) = \frac{d[1-m(y)]}{1-m(y)} \Big/ \frac{dy}{y} \qquad (5)$$

MRP and MREP are invariant respectively to additive and proportional translations of the marginal rates, while RMREP increases (decreases) at all income levels following positive (negative) additive as well as proportional translations of the marginal tax rates.

2.3 Measures of progression based on tax liability and residual income:

Liability progression (LP) proposed by Slitor (1943), and Musgrave and Thin (1948) is defined as the ratio of proportional change in tax liability to proportional change in income:

$$LP(y) = \frac{dt(y)}{t(y)} \Big/ \frac{dy}{y} = \frac{m(y)}{a(y)} \qquad (6)$$

A proportional translation of the average tax rates leaves LP schedule as well as the distribution of tax burden unchanged. This measure is related to average rate elasticity progression `AREP' as LP(y) = AREP(y) + 1[5].

Residual income progression (RIP) proposed by Musgrave and Thin (1948), is defined as the ratio of proportional change in post-tax income to that in pre-tax income:

$$RIP(y) = \frac{d\,[y-t(y)]}{y-t(y)} \Big/ \frac{dy}{y} = \frac{1-m(y)}{1-a(y)} \qquad (7)$$

Residual income progression RIP(y) decreases when the tax becomes more progressive at an income level y. Lambert (1989), with a view to reflect commonly understood positive association between the value of a measure of progression and tax progression redefined residual income progression as:

$$RIP^*(y) = \frac{1}{RIP(y)} = \frac{1-a(y)}{1-m(y)}$$

The magnitude of RIP*(y) increases when the tax becomes more progressive at an income level y.

If the tax structure is such that RIP or RIP* is constant at all levels of income then following a proportional change in pre-tax incomes of all the taxpayers, the redistributive impact of the tax remains unchanged (Jackobsson, 1976).

3. A New Measure: Residual Average Rate Elasticity Progression (RAREP):

Based on changes in marginal tax rate with income, three measures of tax progression, namely, marginal rate progression (MRP), marginal rate elasticity progression (MREP), and residual marginal rate elasticity progression (RMREP) have been developed. Accordingly, three measures of tax progression could have been defined in terms of changes in average tax rate with income. However, only two measures, corresponding to the former two measures have been defined. We define a new measure, corresponding to the latter, in terms of changes in average tax rate with income. The new measure can be defined as the elasticity of residual average tax rate (i.e., one minus the average tax rate) with respect to pre-tax income. It can be expressed as:

$$\text{RAREP}(y) = \frac{d[1-a(y)]}{1-a(y)} \Big/ \frac{dy}{y}$$

$$= -\frac{a(y)}{1-a(y)} \cdot \text{AREP}(y) = \frac{1-m(y)}{1-a(y)} - 1 \ldots\ldots\ (8)$$

A tax structure is progressive if $\text{RAREP}(y) \leq 0$ for all y with strict inequality for some y. Thus a decrease in the value of RAREP(Y) would indicate increase in progression. A positive (negative) proportional or additive translation of the average tax rates decreases (increases) the value of RAREP(y) at all income levels. This means that residual average rate elasticity progression increases (decreases) at all income levels following a positive (negative) additive or proportional translation of the average tax rates. A proof of this is as follows. Let RAREP*(y) and RAREP**(y) denote RAREP(y) after additive (by fraction K) and proportional (by fraction U) translations of the average tax rates respectively.

Now,

$$\text{RAREP*}(y) = \frac{d[1-a(y)-k]}{1-a(y)-k} \Big/ \frac{dy}{y}$$

$$= \frac{1-a(y)}{1-a(y)-k} \cdot \frac{d[1-a(y)]}{1-a(y)} \Big/ \frac{dy}{y}$$

$$= \frac{1-a(y)}{1-a(y)-k} \cdot \text{RAREP}(y)$$

This implies that for $\text{RAREP}(y) \le 0$, i.e., for a progressive tax, $\text{RAREP*}(y) \lesseqgtr \text{RAREP}(y)$ according as $K \gtreqless 0$. Hence the result for both positive and negative additive translations of the average rates.

Also,

$$\text{RAREP**}(y) = \frac{d[1-(1+U)a(y)]}{1-(1+U)a(y)} \Big/ \frac{dy}{y}$$

$$= \frac{(1+U)\ d[1/(1+U)-a(y)]}{1-(1+U)\ a(y)} \Big/ \frac{dy}{y}$$

$$= \frac{(1+U)\ d(1-a(y)}{1-(1+U)a(y)} \Big/ \frac{dy}{y}$$

$$= \frac{(1+U)\ [1-a(y)]}{1-(1+U)\ a(y)} \cdot \frac{d[1-a(y)]}{1-a(y)} \Big/ \frac{dy}{y}$$

$$= \frac{1-(1+U)a(y) + U}{1-(1+U)\ a(y)} \cdot \text{RAREP}(y)$$

This implies that for $\text{RAREP}(Y) \le 0$, $\text{RAREP**}(y) \lesseqgtr \text{RAREP}(y)$ according as $U \gtreqless 0$. Hence the result for both positive and negative proportional translations of the average rates.

From equations 7 & 8 it is clear that RAREP is related to RIP as:

RIP(y) = RAREP(y) + 1[6].

4. Compatibility of Local Measures of Progression

The measures of progression defined in terms of change in marginal tax rate with income are not compatible with those defined in terms of change in average tax rate with income. When the measures in the former category show the tax to be proportional over an income range, the measures in the latter category may show that the tax is progressive. This can be illustrated with a tax function that is linear in income with a nonzero intercept term. With such a tax function, average tax rate rises with income whereas marginal tax rate remains unchanged implying that the measures in the former category would show that the tax is progressive while those in the latter category would suggest that the tax is proportional. For a tax schedule with different marginal tax rates for various income brackets, the measures based on changes in marginal tax rate would show the tax to be proportional within the income brackets whereas those based on average tax rate would suggest that the tax is progressive within the income brackets. Thus the measures - such as average rate progression (ARP or ARP*), residual average rate progression (RARP), average rate elasticity progression (AREP) and residual average rate elasticity progression (RAREP) - are not compatible with any of the measures based on marginal tax rate, namely, marginal rate progression (MRP), residual marginal rate progression (RMRP), marginal rate elasticity progression (MREP), and residual marginal rate elasticity progression.[7]

Among the measures based on average tax rate; average rate progression (ARP or ARP*) and residual average rate progression (RARP or RARP*) differ only with respect to sign. An increase (decrease) in ARP (or ARP*) implies an increase (decrease) in absolute value of RARP (or RARP*). Both the measures - average and residual average rate progression are not compatible with any other measure in this class, i.e., average rate elasticity progression (AREP) and residual average rate elasticity progression (RAREP). Also, these are not found compatible with liability progression (LP) and residual income progression (RIP or RIP*). Kakwani, however, has argued that ARP and LP are compatible. This contention of Kakwani has been shown to be a misconception. Podder (1990) shows that ARP can vary while LP remains unchanged.

Average rate elasticity progression (AREP) is found compatible with liability progression (LP). An increase (decrease) in AREP implies an increase (decrease) in LP and vice versa. Residual average rate elasticity progression (RAREP) is found compatible with residual income progression (RIP or RIP*). An increase (decrease) in RAREP implies an increase (decrease) in RIP or a decrease (increase) in RIP*[8] and vice versa.

Among the measures based on marginal tax rate, marginal rate progression (MRP) and residual marginal rate progression (RMRP) differ only with respect to sign. Both the measures - MRP and RMRP are not compatible with both the other measures in this class, i.e., marginal rate elasticity progression (MREP) and residual marginal rate elasticity progression (RMREP). The latter two measures are also not compatible; RMREP may vary while MREP remains unchanged. For example, a positive (negative) proportional translation of the marginal tax rates leaves MREP unchanged whereas it increases (decreases) RMREP at all levels of income. Further, none of these measures is compatible with liability progression (LP) or residual income progression (RIP or RIP*).

The above discussion on compatibility of contemporary measures of tax progression suggests that, in general, different measures of progression may give rise to different ranking schedules of given tax rate schedules. Therefore a statement about changes in progression or comparison of progression schedules of different tax rate schedules would be useful only if accompanied by definition of the specific measure used. This means that the choice of an appropriate measure of tax progression seems crucial in comparing different tax rate schedules.

Some axiomatic approaches have been proposed for the choice of an appropriate measure of tax progression. The axiomatic approaches are found to have only limited applicability and do not result in the choice of a unique measure of progression.[9] Specific groups of measures seem to reflect on different aspects of a rate schedule. The measures based on changes in residual income or tax rates with pre-tax income seem better suited as indicatives of potential of a rate schedule to redistribute income. The measures based on proportional changes in tax rates or tax liability vis-a-vis proportional changes in pre-tax income appear to be more suitable as indicatives of distribution of tax burden. The measures based on changes in marginal tax rates with income indicate potential of a rate schedule to distort individual choices associated with marginal tax rates such as work effort.[10] The characteristics/properties of some of the measures suggest that these measures can have useful applications in tax design which are explored in the next section.

5. Implications in Tax Design

Growth in per capita nominal incomes of persons may affect inequality in the distribution of pre-tax as well as post-tax income, and redistributive impact of a tax[11]. A policy maker may like to have some control on the effect of income growth on redistributive impact of the tax, if any. Such a control may be built-in into the tax design or exercised through some discretionary change. Some of the local measures of tax progression seem to have important applications in tax design and in taking subsequent policy decisions to achieve the desired effects when income of all the persons change proportionately. Such characteristics of the local measures are explored and applications of these are discussed below.

Characteristics of the relevant measures are stated in the form of propositions and proofs are given. The following notations are used:

S_{yi}, S^*_{yi}	=	Shares of the ith group of taxpayers in post-tax income respectively before-and after a proportional change in pre-tax incomes.
S_{ti}, S^*_{ti}	=	Shares of the ith group of taxpayers in tax burden respectively before-and after a proportional change in pre-tax incomes.
a_i, a	=	Average tax rates respectively of the ith group of taxpayers' and of all the taxpayers.
Y_i, Y	=	Pre-tax incomes respectively of the ith group of taxpayers' and of all the taxpayers.
r	=	A constant representing magnitude of proportional change in pre-tax incomes.
r*	=	A constant representing magnitude of tax progression at all income levels.

Proposition 1: For a given tax structure with constant AREP(y) for all y, a uniform increase (decrease) in pre-tax incomes leaves the distribution of tax burden unchanged and decreases (increases) inequality in the distribution of post-tax income.

Proof: Constant AREP(y), i.e., AREP(y)=r* for all y, means that the average tax rates of all the taxpayers rise (fall) by a proportion rr* following a rise (fall) in their pre-tax incomes by a constant proportion r. So,

$$
\begin{aligned}
S_{ti} &= a_iY_i/aY \\
S^*_{ti} &= a_i(1+rr^*)Y_i(1+r)/a(1+rr^*)Y(1+r) \\
&= a_iY_i/aY \\
&= S_{ti}
\end{aligned}
$$

This shows that a constant proportional change in incomes of all the taxpayers leaves the distribution of tax burden unchanged.

Also,

$$S_{yi} = (1-a_i)Y_i/(1-a)Y$$

$$S^*_{yi} = [\{1-a_i(1+rr^*)\}Y_i(1+r)]/[\{1-a(1+rr^*)\}Y(1+r)]$$

$$= (1-a_i-rr^*a_i)Y_i/(1-a-rr^*a)Y$$

$$\frac{S^*_{yi}}{S_{yi}} = (1-\frac{a_i}{1-a_i}rr^*)/(1-\frac{a}{1-a}rr^*) \qquad (9)$$

For $rr^*>0$ (i.e., for a uniform/proportional increase in pre-tax incomes), $S^*_{yi}/S_{yi} \gtreqless 1$ according as $a_i \lesseqgtr a$ respectively. For $rr^*<0$ (i.e., for a proportional decrease in pre-tax incomes), $S^*_{yi}/S_{yi} \lesseqgtr 1$ according as $a_i \lesseqgtr a$ respectively.

These conditions imply that, for a progressive tax, a proportional increase (decrease) in pre-tax incomes increases (decreases) the shares in post-tax income of the groups of low income taxpayers with group average tax rate less than the global average tax rate and decreases (increases) the shares of the groups of high income taxpayers with group average tax rate higher than the global average tax rate. Thus a uniform increase (decrease) in pre-tax incomes decreases (increases) inequality in the distribution of post-tax income provided the tax structure is such that average rate elasticity progression remains unchanged all along the income scale.

Proposition 2: For a given tax structure with constant LP(y) for all y, a uniform increase (decrease) in pre-tax incomes leaves the distribution of tax burden unchanged and the distribution of post-tax income decreases (increases).

Proof: Constant LP(y), i.e., $LP(y)=r^*$ for all y, means that the tax liability of each of the taxpayers rises (falls) by a proportion rr^* following a rise (fall) in their pre-tax incomes by a constant proportion r. Using the earlier notations, we can write:

$$S_{ti} = a_iY_i/aY$$

$$S^*_{ti} = a_iY_i(1+rr^*)/aY(1+rr^*)$$

$$= S_{ti}$$

This implies that a constant proportional change in pre-tax incomes of all the taxpayers levels the distribution of tax burden unchanged.

Also,

$$S_{yi} = (Y_i - a_iY_i)/(Y - aY)$$

$$S^*_{yi} = [Y_i - a_iY_i(1+rr^*)]/[Y - aY(1+rr^*)]$$

or $$S^*_{yi} = [(1 - \frac{1+rr^*}{1+r} a_i)Y_i \;/\; (1 - \frac{rr^*}{1+r} a)\,Y$$

$$S^*_{yi}/S_{yi} = [(1 - \frac{1+rr^*}{1+r} a_i)\;(1-a)]/[(1 - \frac{rr^*}{1+r} a)\;(1-a_i)] \quad \ldots\ldots\ldots (10)$$

For a progressive tax (i.e., for $r^*>1$); for a proportional increase in pre-tax incomes (i.e., for $r>0$), $S^*_{yi}/S_{yi} \gtreqless 1$ according as $a_i \lesseqgtr a$ respectively; and for a proportional decrease in pre-tax incomes (i.e., for $r<0$), $S^*_{yi}/S_{yi} \lesseqgtr 1$ according as $a_i \lesseqgtr a$. These conditions are the same as those for equation (9). Thus following the analogy of equation (9) it can be stated that a uniform increase (decrease) in pre-tax incomes decreases (increases) inequality in the distribution of post-tax income.

Proposition 3: A tax function of the form $t(y)=\alpha y^{\beta}$ where α and β are parameters has constant AREP(y) as well as constant LP(y) at all income levels.

Proof: For $t(y) = \alpha y^{\beta}$, average tax rate $a(y)$ equals $\alpha y^{\beta-1}$.

$$\text{AREP}(Y) = \frac{da(y)/a(y)}{dy/y}$$

$$= \frac{y}{\alpha y^{\beta-1}} \quad \frac{d(\alpha y^{\beta-1})}{dy}$$

$$= \beta-1$$

Means that AREP(y) does not depend on income level y. Hence it remains constant at all income levels.

Similarly,

$$\text{LP}(y) = \frac{dt(y)/t(y)}{dy/y}$$

$$= \frac{y}{\alpha y^{\beta}} \cdot \frac{d(\alpha y^{\beta})}{dy}$$

$$= \beta$$

Hence LP(y) remains constant at all income levels.

A uniform increase (decrease) in pre-tax incomes of all the taxpayers leaves inequality in the distribution of pre-tax income unchanged, and propositions 1 & 2 suggest that inequality in the distribution of post-tax income declines (rises) provided the tax structure is of constant average rate elasticity progression or constant liability progression at all income levels. It implies that the redistributive impact defined as the difference between indices of inequality in the distributions of pre- and post-tax incomes would rise (decline) following a uniform rise (decline) in pre-tax incomes of all the taxpayers. Proposition 3 suggests that for a tax function of the form $t(y)=\alpha y^{\beta}$, AREP(y) as well as LP(y) remain constant at all income levels. So, for such a tax function/structure, redistributive impact of the tax rises (declines) and distribution of tax burden remains unchanged following a proportional increase(decrease) in incomes of all the taxpayers. This result seems to have important policy implications as to the tax design. If it is desirable that in a period of inflation (deflation) that uniformly increases (decreases) pre-tax incomes of all the taxpayers, redistributive impact of the tax should rise (decline) then the tax design should be such that average rate elasticity progression or liability progression remains unchanged all along the income scale. This condition is satisfied by a tax function of the form, $t(y)=\alpha y^{\beta}$. Further such a rise(decline) in the redistributive impact can be controlled or even fully neutralised through an appropriate translation of average tax rates of all the taxpayers. For complete neutralisation, average tax rates of all the taxpayers should be deflated by the factor $[1+g(\beta-1)]$, i.e., by the factor $(1+gp)$, where g is the growth rate of income and p is the magnitude of average rate elasticity progression or that of liability progression reduced by one. For partial neutralisation, the average tax rates should be deflated by a factor $(1+X)$ where $0 < X < gp$.

Proposition 4: For a given progressive tax structure with constant RAREP(y) for all y, a uniform increase (decrease) in pre-tax incomes of all the taxpayers leaves the distribution of post-tax income unchanged, and the distribution of tax burden changes against (in favour of) the low income taxpayers.

Proof: For constant RAREP(y), i.e., for RAREP(y) = r* for all y, residual average tax rate [1-a(y)] of each of the taxpayers would change by a proportion rr* following a proportional change in their pre-tax incomes by a constant fraction r. Using the earlier notations, we can write:

$$S_{yi} = (1-a_i)Y_i/(1-a)Y$$

$$S^*_{yi} = (1-a_i)(1+rr^*)Y_i/(1-a)(1+rr^*)Y$$

$$= S_{yi}$$

This implies that a constant proportional change in pre-tax incomes of all the taxpayers leaves the distribution of post-tax income unchanged provided the tax structure is such that residual average rate elasticity progression remains unchanged all along the income scale.

Also,

$$S_{ti} = a_i Y_i / aY$$

$$S^*_{ti} = [a_i(1+rr^*)-rr^*]Y_i/[a(1+rr^*)-rr^*]Y$$

$$S^*_{ti}/S_{ti} = \frac{1+(1-1/a_i)rr^*}{1+(1-1/a)rr^*} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

It is noteworthy that for a progressive tax, r^* should be negative. Therefore, a uniform/proportional increase (decrease) in pre-tax incomes of all the taxpayers would mean $rr^*<0$ (>0). Equation (11) suggests that for $rr^*<0$, $S^*_{ti}/S_{ti} \lesseqgtr 1$ according as $a_i \gtreqless a$ respectively, and for $rr^*>0$, $S^*_{ti}/S_{ti} \gtreqless 1$ according as $a_i \gtreqless a$ respectively.

These conditions imply that, for a progressive tax, a proportional increases (decreases) in pre-tax incomes of all the taxpayers decreases (increases) the shares in tax of the groups of high income taxpayers with group average tax rate greater than the global average tax rate and increases (decreases) the shares of the groups of low income taxpayers with group average tax rate lower than the global average tax rate. Thus following a uniform increase (decrease) in pre-tax incomes, distribution of tax burden changes against (in favour of) the low income taxpayers.

Proposition 5: A tax function of the form, $t(y)=y-\tau y^{\delta}$, where τ and δ are parameters, has constant RAREP(y) as well as constant RIP(y) at all income levels.

Proof: For $t(y) = y - \tau y^{\delta}$, average tax rate $a(y)$ equals $1 - \tau y^{\delta-1}$

$$RAREP(y) = \frac{d[1-a(y)]}{1-a(y)} / \frac{dy}{y}$$

$$= \frac{d(\tau y^{\delta-1})}{\tau y^{\delta-1}} / \frac{dy}{y}$$

$$= \delta - 1$$

This implies that RAREP(Y) does not depend on income level -. Hence it remains constant at all income levels.

Similarly,

$$RIP(y) = \frac{[(1-a(y))y]}{([1-a(y)]y} / \frac{dy}{y}$$

$$= \frac{d(\tau y^{\delta-1}.y)}{\tau y^{\delta-1}.y} / \frac{dy}{y}$$

$$= \frac{d(\tau y^{\delta})}{\tau y^{\delta}} / \frac{dy}{y} = \delta$$

Hence RIP(y) remains constant at all income levels.

Proposition 4 suggests that the distribution of post-tax income is neutral to equi-proportional changes in pre-tax incomes of all the taxpayers provided the tax structure is of constant residual average rate elasticity progression. Jackobsson (1976) has shown that the distribution of post-tax income is neutral to equi-proportional changes in pre-tax incomes of all the taxpayers provided the tax structure is of constant residual income progression. This means that a proportional change in pre-tax incomes of all the taxpayers does not affect redistributive impact of the tax provided the tax structure is of constant residual average rate elasticity progression or constant residual income progression at all income levels. Proposition 5 suggests that for a tax function of the form, $t(y)=y-\tau y^{\delta}$, RAREP(y) as well as RIP(y) remain constant at all income levels. So, for such a tax function/ structure, a uniform increase (decrease) in incomes of all the taxpayers does not affect its redistributive impact, and changes the distribution of tax burden against (in favour of) the low income taxpayers. An implication of this result in tax design is that if it is desired that in a period of inflation (deflation) that uniformly increases (decreases) pre-tax incomes of all the taxpayers, redistributive impact of the tax should remain unchanged and the distribution of tax burden should change against (in favour of) the low income taxpayers then the tax design should be such that residual average rate elasticity progression or residual income progression remains unchanged all along the income scale. This condition is satisfied by a tax function of the form, $t(y)=y-\tau y^{\delta}$.

6. Conclusion

The local measures of tax progression are found to have important applications in tax design and subsequent policy decisions to achieve the desired effects. A new measure of tax progression - residual average rate elasticity progression' is introduced that has applications in tax design. The study suggests that, for a progressive tax, if it is desirable that the redistributive impact of the tax should remain unaltered following an equi-proportional change in pre-tax incomes of all the taxpayers then the tax function/structure should be so designed that residual average rate elasticity progression or residual income progression remains unchanged all along the income scale - a condition that is satisfied by a tax function of the form, $t(y) = y-\tau y^{\delta}$ where y is income level, and τ & δ are parameters. On the other hand, if it is desirable that a proportional increase (decrease) in incomes of all the taxpayers should increase (decrease) redistributive impact and leave the distribution of tax burden unchanged then the tax function/structure should be so designed that average rate elasticity progression or liability progression remains unchanged all along the income scale - a condition that is satisfied by a tax function of the form $t(y)= \alpha y^{\beta}$, where α and β are parameters. Further, the resultant increase or decrease in the redistributive impact can be partly or fully neutralised through an appropriate translation of the average tax rates of all the taxpayers.

TABLE 1

Salient Features of Local Measures of Tax Progression

S.No.	Measure of Progression	Tax is progressive, proportional or regressive according as the measure is	Rise(↑) or fall(↓) in in progression due to a constant increase (decrease) in tax rates at all income levels: Proportional change	Percentage point change
	(1)	(2)	(3)	(4)
Earlier Measure				
1.	Average rate progression (ARP)	≥ 0 $<$	↑ (↓)	neutral
2.	Average rate elasticity progression (AREP)	≥ 0 $<$	neutral	↓ (↑)
3.	Marginal rate progression (MRP)	≥ 0 $<$	↑ (↓)	neutral
4.	Marginal rate elasticity progression (MREP)	≥ 0 $<$	neutral	↓ (↑)
5.	Residual marginal rate elasticity progression (RMREP)	≤ 0 $>$	↑ (↓)	↑ (↓)
6.	Liability progression (LP)	≥ 1 $<$	neutral	↓ (↑)
7.	Residual income progression (RIP)	≤ 1 $>$	↑ (↓)	↑ (↓)
A New Measure				
8.	Residual average rate elasticity progression (RAREP)	≤ 0 $>$	↑ (↓)	↑ (↓)

NOTES

1 A measure advocated by Baum (1987), i.e., relative income share progression (RISP) combines the characteristics of both local and global measures of progression. Discussion of this measure is beyond the scope of this paper.

2 For a lucid discussion on global measures of progression see, for example, Kiefer (1984) and Pfahler (1987).

3 An additive translation of average tax rates a(y) is defined as a(y)+c, where c is a constant fraction. For c>0 (c<0) it is called positive (negative) additive translation.

4 A proportional translation of average tax rates a(y) is defined as (1+c).a(y), where c is a constant fraction. For c>0 (c<0) it is called positive (negative) proportional translation.

5 Although LP and AREP differ only by a constant, these depict different approaches to measuring progression. It is noteworthy that a change in the tax schedule that leads to a proportional change in LP at all income levels will not result in proportional change in AREP.

6 Although RIP and RAREP differ only by a constant, these like LP and AREP depict different approaches to measuring progression. It is noteworthy that a change in the tax schedule that leads to proportional change in RIP at all income levels will not result in proportional change in RAREP.

7 For a continuously rising or falling tax rate, i.e., for marginal tax rate that rises or falls at all levels of income, average rate progression and marginal rate progression; and average rate elasticity progression and marginal rate elasticity progression can be said to be pair-wise compatible.

8 Note that RIP(y)<1 indicates that the tax is progressive at income level y whereas RIP*(y)>1 indicates the same.

9 See, for example, Aggarwal (1980) and Kakwani (1980).

10 For an extensive discussion on suitability of specific groups of measures see, for example, Aggarwal (1980).

11 See, for example, Jakobsson (1976), Hutton and Lambert (1982), and Pechman (1982).

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