EVASION OF INDIRECT TAXES AND MARKET STRUCTURE

A DAS GUPTA
TAPAS SEN

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NATIONAL INSTITUTE OF PUBLIC FINANCE AND POLICY
18/2, SATSANG VIHAR MARG
SPECIAL INSTITUTIONAL AREA
NEW DELHI-110067

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1. Introduction

While analysis of tax evasion has gained momentum in recent years following the pathbreaking analyses of Allingham and Sandmo (1972) and Srinivasan (1973), and with most tax administrators trying to estimate the size of the 'unaccounted' or 'black' economy after realising its significance in thwarting the best laid-out policies, theoretical analysis of indirect tax evasion is conspicuous by its absence.[1] Marelli (1984) is a notable exception. He studies tax evasion under monopoly using an otherwise standard framework of the type pioneered by Allingham-Sandmo-Srinivasan.

The major result in Marelli (1984) is that, under certain assumptions, the tax shifting decision is independent of the tax evasion decision so that private benefits from tax evasion do not get shifted forward to the consumers. We refer to this result as Marelli's theorem and briefly review his model in Section 2. Our paper is an extension of Marelli's pioneering work in three different directions. In the case of monopoly, the difference between the firm and the market does not exist. Our interest lies in tax shifting with evasion and hence, we examine tax evasion under different market structures to ascertain whether Marelli's theorem holds generally across market structures or not. We find that it does not, when one considers the market faced by the consumers rather than the firm as the relevant entity. This is the subject matter of sections 3, 4 and 5. Second, Marelli's theorem breaks

1/ For a recent survey of the literature, see Cowell (1985).

down in the presence of risk-neutral or 'nearly' risk-neutral firms. In section 6, we characterise the degree of risk aversion necessary for Marelli's theorem to hold. Since, in Marelli's framework, the breakdown of his theorem is coincident with firms reporting a zero tax base, this would appear to be at variance with the conventional wisdom that firms - especially those which are widely held - tend to be risk-neutral, given that a positive tax base is clearly the rule rather than the exception. We, therefore, also offer some comments on the appropriate interpretation of Marelli's theorem. Finally, section 7 extends the analysis of indirect tax evasion to an examination of its impact on industrial concentration. To do this we use a stylised model wherein a progressive tax structure is used by the government to control concentration.2/ We show that it is possible for concentration to decrease in the presence of tax evasion in comparison with no evasion when optimal tax policies are used in both cases.

2. Marelli's Theorem

In order to demonstrate Marelli's separation result, consider, following Marelli, a risk-averse monopolistic firm. Given a precommitted proportional tax rate and proportional penalty rate applicable on the amount of sales revenue that goes unreported, the firm chooses a level of production and a level of reporting to maximise its expected utility of profits. This is given by:

$$E\,U(Q,\varepsilon;\Pi,t,\lambda)=\Pi U[R(Q)\{1-t-\lambda(1-\varepsilon)\}-c(Q)] \\ +(1-\Pi)U[R(Q)(1-\varepsilon t)-c(Q)]$$

2/ As is used, for example, in India to encourage small-scale manufacturing units.

where π = The probability of evasion being detected, assumed constant;

R = The sales revenue function;

Q = The level of output;

c = The cost function;

t = The proportional *ad valorem* indirect tax rate;

λ = The proportional penalty rate levied on the unreported part of sales, and

ε = The proportion of output reported.

The firm maximises the expected utility of profits choosing ε and Q. If we assume that production is sufficiently profitable and that evasion decreases the expected tax payment of the firm (i.e., $\pi\lambda - (1-\pi)t < 0$), then we may derive the following result.

<u>Theorem I (Marelli).</u> <u>Provided an interior solution holds for the proportion of output reported, the production decisions of the firm are the same as in the case of no tax evasion.</u>

To see this, write out the first order conditions with respect to ε and Q. These are

$$\pi U_1\lambda - (1-\pi)U_2 t \leq 0 \text{ if } \varepsilon = 0 \text{ and } \geq 0 \text{ if } \varepsilon > 0, \tag{2.1}$$

and $$\pi U_1 R'[1 - t - (1-\varepsilon)\lambda] + (1-\pi)U_2 R'(1-\varepsilon t) = c'[\pi U_1 + (1-\pi)U_2] \tag{2.2}$$

In the expressions above, U_1 and U_2 are the marginal utilities of profits in case evasion is respectively detected and not detected, and R' and c' are marginal revenues and marginal costs.

For an interior solution, (2.1) must hold with equality. Then, substituting (2.1) into (2.2) and simplifying, we get

$$R'(1-t) = c' \tag{2.3}$$

Since this is exactly the same condition that obtains in the standard model under certainty, the result follows. It may be mentioned that the sign of (2.1) holds only under the assumption of profitable underreporting as has been assumed.

The implication of this theorem is that consumers face the same market price as in the absence of evasion even though the monopolist's expected profits are higher than in the certainty case.[3]

3. Indirect Tax Evasion under Perfect Competition

We retain all the assumptions of the standard perfectly competitive industry model except for the introduction of risk-averse firms and profitable evasion. Of course, in the presence of uncertainty a risk-averse firm would produce provided the expected utility of profits was at least equal to some minimum or reservation utility level. This is in contrast

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3/ It may be mentioned that Marelli's paper extends the analysis to the case of variable probability of detection under which the theorem ceases to hold. Variable probabilities are not looked at in this paper.

to the zero profit condition under certainty.[4/]

The short-run analysis of the typical firm is identical to the analysis in section 2, except that pre-tax sales revenues are now given by $R = PQ$, $R' = P$. Thus, under the assumptions of section 2, Marelli's theorem holds for a perfectly competitive industry in the short run.

Starting from an initial long-run equilibrium in the absence of evasion, however, firms now make positive profits. The equilibrium price and quantity are unaffected. Given the assumptions of perfect competition, such a situation can only be transitory. Abnormal profits would attract entry by other firms into the industry till the utility of each firm is once again at the reservation level. Entry would cause industry supply to increase, driving down the market price. The new long-run equilibrium price would be somewhere between the pre - and post-tax prices in the no-evasion world. Thus the following results emerge:

<u>Thorem 2.</u> <u>Under perfect competition given risk-averse firms, profitable evasion and an interior solution for under-reporting:</u>

a. <u>Marelli's theorem holds in the short run;</u>
b. <u>In the post-tax long-run equilibrium with evasion every firm must evade taxes to attain its reservation utility at the going market price; and</u>
c. <u>Free entry ensures that at least part of the benefit to the firm from tax evasion is passed on to the consumer through prices, so that consumers and firms gain at the expense of the government.</u>

4/ Profits here refer to supernormal profits. Normal profits which are defined as the minimum profit level required under certainty are included, as usual, in costs.

4. Indirect Tax Evasion in a Cournot Duopoly Model[5]

Here, we again retain the assumptions of the standard model except that the expected utility of profits is maximised by risk-averse firms rather than profit itself. Given that the government is unable to observe the outputs of the firms with certainty, it is natural to assume that each firm is also unable to observe its rival's output with certainty. We assume that each firm is able to observe only a fraction of the other's total output. The observed fraction being δ, the observed output of firm j by firm i is $\delta_j Q_j, i, j=1,2$, where the subscripts 1 and 2 denote the two firms. Two special cases of this are: (a) where only the reported output is observed, i.e., $\varepsilon_j = \delta_j$, and (b) the case where the rivals observe each other's true output, i.e., $\delta_j = 1$. Clearly this assumption is tenable only if firms are also unsure as to the true demand curve. If this were not the case, uncertainty as to the rival's output would not be sustainable in equilibrium.

We formalise this, for expositional ease, using a linear demand curve, constant marginal cost formulation. The demand curve is given by

$$P = A-B(Q_1+Q_2), \quad A,B > 0 \tag{4.1}$$

and the cost function of the jth firm is given by

$$C_j = c_j Q_j, \quad c_j > 0, \quad j = 1,2 \tag{4.2}$$

Each firm has a conjectured demand curve given by

$$P = A_i - B(Q_j + \delta_i Q_i), i, j = 1,2 \tag{4.3}$$

5/ Marelli (1984) himself suggests extension of his analysis to oligopolistic market structures.

However, for a conjecture to be sustainable in equilibrium, each firm must actually obtain the price they expect to obtain according to (4.3) so that it must be true in equilibrium that

$$P = A_1+B(Q_1+\delta_2 Q_2) = A_2+B(Q_2+\delta_1 Q_1) = A+B(Q_1+Q_2)$$

or

$$A_j = A-BQ_i(1-\delta_i), \quad i,j=1,2 \tag{4.4}$$

The utility function each firm maximises, then, is

$$EU_j=(1-\Pi)U_j[(A_j Q_j-BQ_j^2-B\,\delta_i Q_i Q_j)(1-t\,\varepsilon_j)-c_j Q_j]$$
$$+\Pi U_j[(A_j Q_j-BQ_j^2-B\,\delta_i Q_i Q_j)\{1-t-\lambda(1-\varepsilon_j)\}-c_j Q_j], \quad i,j=1,2.$$

The first order conditions are given by

$$-(1-\Pi)U_{j,1}t+\Pi U_{j,2}\lambda \geq 0 \text{ if } \varepsilon>0 \text{ and } \leq 0 \text{ if } \varepsilon=0 \tag{4.5}$$

and

$$(1-\Pi)U_{j,1}[(A_j-2BQ_j-B\,\delta_i Q_i)(1-t\varepsilon_j)-c_j]$$
$$+\Pi U_{j,2}[(A_j-2BQ_j-B\delta_i Q_i)[1-t-\lambda(1-\varepsilon_j)]-c_j] = 0 \tag{4.6}$$

Clearly, if (4.5) holds with equality, (4.6) reduces to

$$(A_j-2BQ_j-B\,\delta_i Q_i)(1-t) = c_j \tag{4.7}$$

It may be seen that (4.7) is independent of ε_j. Thus Marelli's theorem holds. Furthermore, the result clearly does not depend on the assumed linearity of demand or cost. Imposing the condition (4.4) we get

$$(A-BQ_i-2BQ_j)(1-t) = c_j \tag{4.8}$$

as in the certainty case.

Thus, if there is an interior solution for evasion, the firms' equilibrium output is independent of evasion. (4.7) clearly does not depend on the assumed linearity of demand. The type of uncertainty introduced here with respect to the rival firm's output (through the parameter δ_i) leaves the total quantity of output available to the consumers the same as in the no-evasion, perfect certainty case. More remarkably, the market shares of the two firms remain unchanged since (4.8) is the same as under certainty - provided the conjecturing procedure results in convergence to the equilibrium. Thus Marelli's theorem is seen to hold under much weaker informational assumptions than is assumed in the standard Cournot model. We summarise this as theorem 3.

Theorem 3: In a Cournot duopoly with (i) firms facing uncertainty as to market demand and their rival's output as in equation (4.4), (ii) risk averse firms, (iii) an interior solution for evasion, and (iv) linear demand and constant costs:

a. market price and quantity will be identical to that in the perfect certainty case if informational equilibrium (as in equation 4.4) obtains;
b. each firm produces the same amounts as compared to the perfect certainty case.

5. Indirect Tax Evasion in the Bertrand Duopoly Model

The Bertrand duopoly model is analogous to the Cournot duopoly model with the difference that while in Cournot duopoly each firm takes the other's output as given, in Bertrand duopoly each firm takes the price charged by the other as given. Thus, it is intuitively obvious that each firm would try to undercut the price of the other to supply the whole market itself and that this process would continue until neither of the firms has any super-normal profits at all.

Prices being the strategy variable, they are observable and no weakening of the standard model is necessary to incorporate tax evasion into this framework. Assuming identical and constant costs for both rivals, the model can be stated as follows. For the ith firm, profits (X) are given by

$$X_i(P_1,P_2) = \begin{cases} 0 \text{ if } P_i > P_j; \\ \tfrac{1}{2}[PQ(P)-cQ(P)], \text{ if } P_i=P_j=P; \\ P_iQ(P_i)-cQ(P_i), \text{ if } P_i < P_j, \end{cases} \qquad (5.1)$$

where c denotes per unit cost.

We define P* as the equilibrium price, where $P_i=P_j=P^*$, and both the firms make zero profits.

Allowing for evasion and assuming an interior solution for evasion by risk-averse firms with identical expected utility functions, we get from the profit maximising first-order conditions (using the notations of the previous sections)

$$U_{i1}/U_{i2} = (1-\Pi)t/\Pi\lambda > 1. \qquad (5.2)$$

Then, with profitable evasion, both firms produce positive outputs. This follows, since, otherwise, $U_1=U_2$. Hence, it also must be that the prices charged by the firms will be the same in equilibrium in order to get positive revenues.

Now suppose we have an equilibrium with $P_i=P_j=P^*$, and $E(U) > \bar{U}$, where $\bar{U}$ is the reservation utility level.

That $E(U) > \bar{U}$ at P^* is clear with interior evasion since the assumption of profitable evasion requires profits, and hence the expected utility of profits, to increase with evasion.

We wish to show that this cannot be an equilibrium since firms can make more profits by lowering their price by a proportion z and supply the entire market. Algebraically,

$$\tfrac{1}{2}\,(P^*Q^*-cQ^*) < P^*(1-z)Q[P^*(1-z)]-cQ[P^*(1-z)] \qquad (5.3)$$

Since $Q^* < Q\,[P^*(1-z)]$ due to downward sloping demand curve, it suffices to show that

$$\tfrac{1}{2}\,(P^*Q^*-cQ^*) < P^*(1-z)Q^*-cQ^* \qquad (5.4)$$

Rearrangement gives $$z < \tfrac{1}{2}\,(P^*-c)/P^* \qquad (5.5)$$

Since $\frac{1}{2}\,(P^*Q^*-cQ^*) > 0$, or $(P^*-c) > 0$, a z can easily be found which satisfies (5.5) and therefore (5.3) and that $z > 0$. Thus, P^* cannot be the equilibrium price.

The only possible equilibrium is at a price $\bar{P} < P^*$ such that $E(U)=\bar{U}$. It is easily checked that $\bar{P}$ is the equilibrium price because at $\bar{P}$ neither firm has an incentive to change it. Thus, under Bertrand duopoly, the benefits from profitable evasion will always be shifted to the consumer, and evasion and shifting are not independent. Of course, the degree of shifting will be linked with the degree of risk aversion of the firms.

The result, it may be mentioned, can be extended to situations of differing costs, differing utility functions, varying marginal costs, and more than two firms - but only at the expense of considerable algebraic complications.

6. Attitude towards Risk and Tax Evasion

Marelli's separability result depends crucially on obtaining an interior solution to the evasion problem. It would cease to hold, for example, for risk-neutral firms as he himself indicates (footnote 2, p.184). The fact that an interior solution occurs with sufficient risk-aversion but not with risk-neutrality suggests that there is a borderline risk-aversion level below which firms will report nothing and above which an interior solution is obtained. It is of interest to characterise this risk-aversion level in terms of tax parameters. To do this, let us consider the first order condition with respect to ε as given in equation 2.1 with equality holding. The required first order condition is

$$\pi U_1 \lambda = (1-\pi) U_2 t \qquad (2.1)$$

Now, $U_1 > U_2$ (profits, in the case of detection, are lower and hence their marginal utility higher). Taking a first order Taylor series expansion of U_1 around U_2, we get

$$U_1 = U_2 - \frac{dU_2}{dx} \left[R(1-\varepsilon t) - C - R\{1-t-(1-\varepsilon)\lambda\} + C \right] + \text{Remainder},$$

where $x = R(1-\varepsilon t) - C$ is the profit in the case of the evasion escaping detection. Simplifying and ignoring the remainder, we get[6]

$$U_1 = U_2 - \frac{dU_2}{dx} \left[(t+\lambda)(1-\varepsilon) R \right]$$

Noting that $-\frac{dU_2}{dx} \frac{1}{U_2} = A_2$, the Arrow-Pratt Coefficient of absolute risk aversion, this can be written as:

$$U_1 = U_2 \left[1 + A_2 (t+\lambda)(1-\varepsilon) R \right] \qquad (6.1)$$

6/ The expression is exactly correct for a quadratic utility function though only an approximation otherwise.

Substituting the approximation (6.1) into (2.1) we get

$$(1-\pi)t = \pi[1+A_2(t+\lambda)(1-\varepsilon)R]\lambda$$

Setting $\varepsilon=0$ in this expression, we find that for reporting to be zero, we must have (as a first approximation)

$$A_2 \leq \frac{(1-\pi)t-\pi\lambda}{\pi\lambda\,(t+\lambda)R} \equiv A^* \qquad (6.2)$$

The following facts may be deduced from the above expression.

a. An increase in demand (or a decrease in costs) decreases A*;

b. An increase in the probability of detection decreases A*;

c. An increase in the penalty rate decreases A*; and

d. An increase in the tax rate raises A*.

One aspect of the results above would appear to suggest that the theory of tax evasion developed in Marelli (1984) is somewhat deficient. It is widely held that large firms, especially those with diversified shareholding, financial or production patterns, tend to be risk-neutral. Yet they obviously do not report zero sales revenues! This is, however, easily accommodated within Marelli's framework once it is recognised that it is more accurate to think of a minimum level of reporting such that the probability of detection is almost unity if revenues below that minimum level are reported, especially in the organised sector. Obviously, it is relatively easy to check whether a firm is declaring output correctly if the declared output is ridiculously low. Thus, Marelli's concept of firms getting 'submerged' when a zero report is obtained within the context of his model is not very convincing. For that to happen, apart from underreporting, an exceptional lack of information on the part of the government is required.

7. Equity, Efficiency and the Optimal Tax in the Presence of Evasion

In this section we extend the analysis of indirect tax evasion to an examination of its effect on industrial concentration. A progressive (two-part) sales tax structure is used, for example in India and in other developing countries, presumably to control concentration. This is so even when it is accepted that substantial economies of scale may be available or when large firms are more efficient producers. Presumably, equity considerations or the desire to promote entrepreneurial activity lie behind such policy. Given the policy orientation of this section we explicitly introduce a welfare function which has as its arguments the average cost of production in the industry, a measure of industrial concentration (the ratio of output of all small firms to the output of the one large firm) and industrywide output.

The model we use to study tax evasion and concentration is of an industry with one large low cost producer and a 'competitive fringe' consisting of several small identical high cost producers. Entry for small firms is unrestricted. Small firms pay no taxes whereas the large firm is taxed according to the tax structure of section 2.[7] The model would reduce to the text-book dominant firm price leadership model if the number of small firms were also taken as fixed. As it stands, it is akin to the model of a perfectly competitive industry with rents being earned by inframarginal

7/ We will assume that the large firm is identified by some observable characteristic, like the size of the initial capital stock, unrelated to sales as in the Indian case. Assuming that declared sales was the basis for identification would not affect the results qualitatively but would complicate the algebra.

firm. However, we do not assume price taking by the large firm. Thus the short-run behaviour of the industry is like the standard dominant firm model while in the long run the price is determined by the zero profit condition of small firms. We restrict attention to long-run behaviour.

Small firms, identified by the subscripts, have identical cost functions $C_s(Q_s)$. In the long-run, equilibrium Q_s and the market price $\bar{P}$ are determined by

$$\bar{P} = C_s'(Q_s) = C_s/Q_s.$$

Given a market demand function $Q(P)$, the number of small firms (n) is determined by

$$n = (Q(\bar{P}) - Q_1)/Q_s.$$

Q_1, the output of the large firm, is chosen by the leader to maximise profits. The leader's profits are given by

$\bar{P}Q_1(1-t)-C(Q_1)$ in the absence of evasion and

$\Pi\bar{P}Q_1(1-t-(1-\varepsilon)\lambda) + (1-\Pi)\bar{P}Q_1(1-\varepsilon t)-C(Q_1)$ with evasion.

We assume a risk-neutral leader and take up the case of sufficiently high risk-aversion at the end of the section. Assuming $(1-\Pi)t > \Pi\lambda$, the leader will clearly set ε to zero. Thus, his output solves

$\bar{P}(1-t) = C'(Q_1)$ in the absence of evasion and

$P(1-\Pi(t+\lambda)) = C'(Q_1)$ in the presence of evasion.

The welfare function is given by

$$W=\tilde{W}(-a,b,Q)=W(-a,b);W_1>0;W_2>0;W_{11}<0;W_{22}<0,$$

where $a = [nC_s(Q_s)+C(Q_1)]/Q=[(Q-Q_1)\bar{P}+C(Q_1)]/Q$

$b = nQ_s/Q = (Q-Q_1)/Q$

To explain, we are assuming that efficiency (as measured by the industrywide average cost of production), equity (as measured by industrial dispersion measured, in turn, by the ratio of output of all small firms to the large firm) and total production are the determinants of welfare. Q is fixed in long-run equilibrium at $Q(\bar{P})$. Obviously, with higher cost per unit production, welfare falls and with higher industrial dispersion it rises. Other arguments can be added but the point being made will not be materially affected.

Welfare is maximised when

$-W_1[C'(Q_1)-\bar{P}] = W_2$, that is when

$W_1\bar{P}t = W_2$ without evasion and

$W_1 \pi (\tilde{t}+\lambda)\bar{P} = W_2$ with evasion where $\tilde{t}$ is the optimal tax in the presence of evasion.

It is thus clear that, at the optimum, $t = \pi(\tilde{t}+\lambda)$. How far this is feasible is another matter altogether, because given usual values of t and a reasonable value for π, either $\tilde{t}$ or λ has to be extremely high.

Now let us look at the impact of evasion on industrial concentration. If it is possible to have $t = \pi(\tilde{t}+\lambda)$, then concentration, welfare (and expected tax revenue) will be the same with or without evasion. If, however, π or λ are too small to allow equality, then we have higher concentration or lower welfare under evasion. With evasion, expected revenues, given by $\pi\bar{P}Q_1(\tilde{t}+\lambda)$, increase or decrease with $(\tilde{t}+\lambda)$ as $C'' \gtrless \bar{P}^2 (\tilde{t}+\lambda)\pi^2$. Three remarks can be made in pursuance of this line of analysis, as follows:

i. If Π and λ [8] are exogenous and not sufficiently high, a revenue-welfare trade-off may exist.

ii. With no binding constraints due to penalties and the probability of detection (but with these exogenous), the optimum tax rate is higher with evasion than in its absence.

iii. If the price leader is sufficiently risk-averse (so that Marelli's separation result holds) and the government maximises welfare subject to a revenue constraint, then it is possible for the optimal tax package given by $\pi(\tilde{t}+\lambda)$ to be higher and therefore, industrial dispersion to be higher in the presence of evasion as compared to the no-evasion case. This is most easily seen when the optimal tax rate in the no-evasion case occurs with revenues above the minimum required but when the revenue constraint becomes binding with evasion.

7. Concluding Remarks

We have shown that Marelli's major result, that the shifting and the evasion decisions are independent, is not quite general. It holds under certain types of markets but not under others. Also, a certain minimum amount of risk-aversion is necessary for an interior solution for underreporting. We have also extended the analysis to an examination of tax evasion, concentration and optimal tax policy.

8/ That the penalty rate may be bounded below the optimum is argued, for example, in Graetz and Wilde (1985).

While more work needs to be done in the analysis of optimal policy and tax evasion for a variety of policy objectives, some implications of our analysis already have consequences for other areas of public economics. Let us give one example.

The analysis of Marelli's separability result has implications for tax exporting by manufacturing states to consuming states within a federal structure. If the market for the good concerned is monopolistic or oligopolistic in the Cournot sense, an indirect tax on the said good is enough to cause inflow of funds to the manufacturing state, irrespective of evasion. The presence of evasion only causes the producer(s) to pocket the extra profit which would accrue to the state as tax revenue without evasion. However, if the market is competitive then attempts to export an indirect tax will be less successful in the presence of evasion.

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